



Dedicated Attacks on Popular Hash Functions

Christian Rechberger



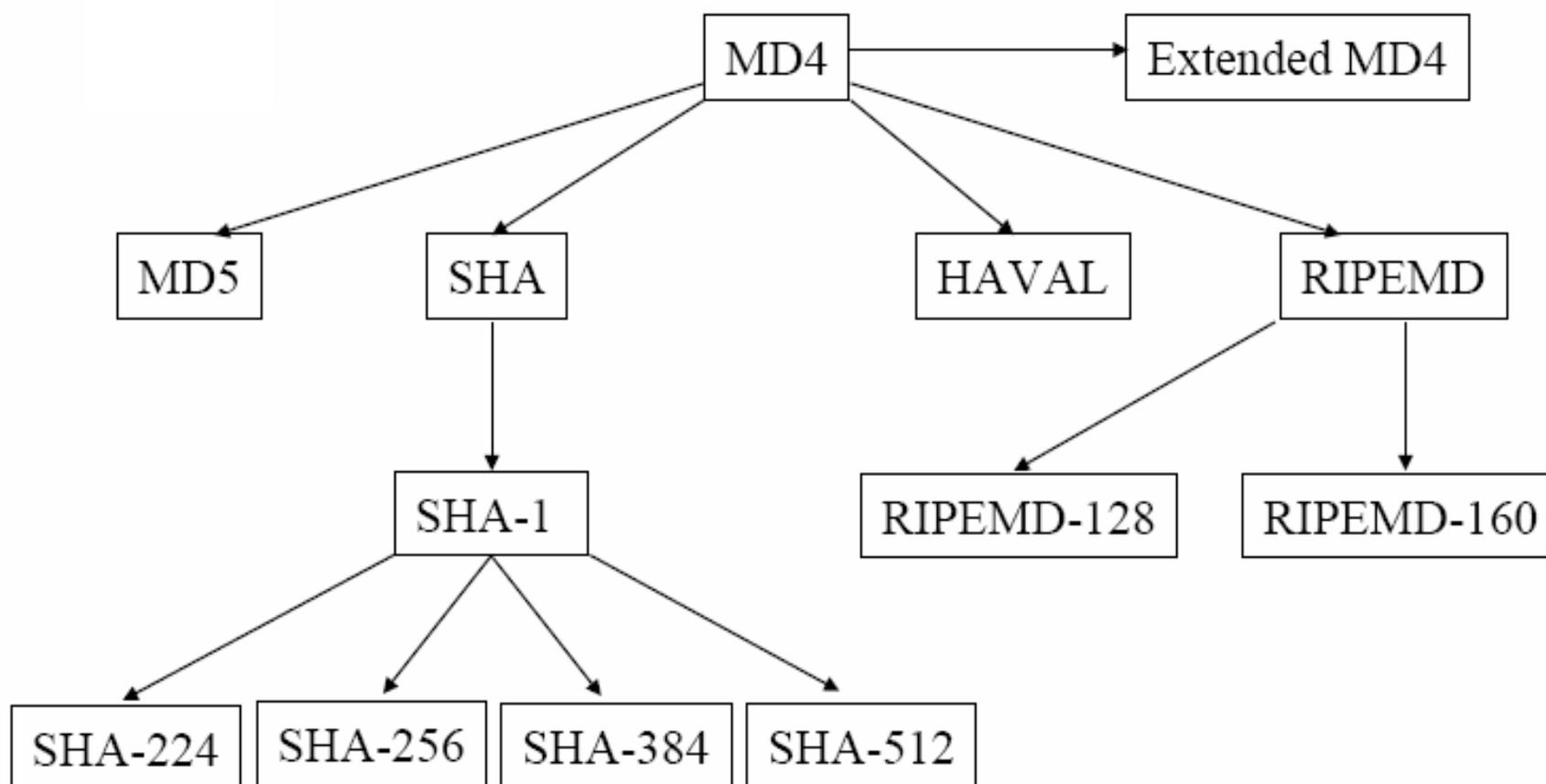
ECRYPT Summer School, May 02, 2007

***Institute for Applied Information Processing
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***Faculty of Computer Science
Graz University of Technology***

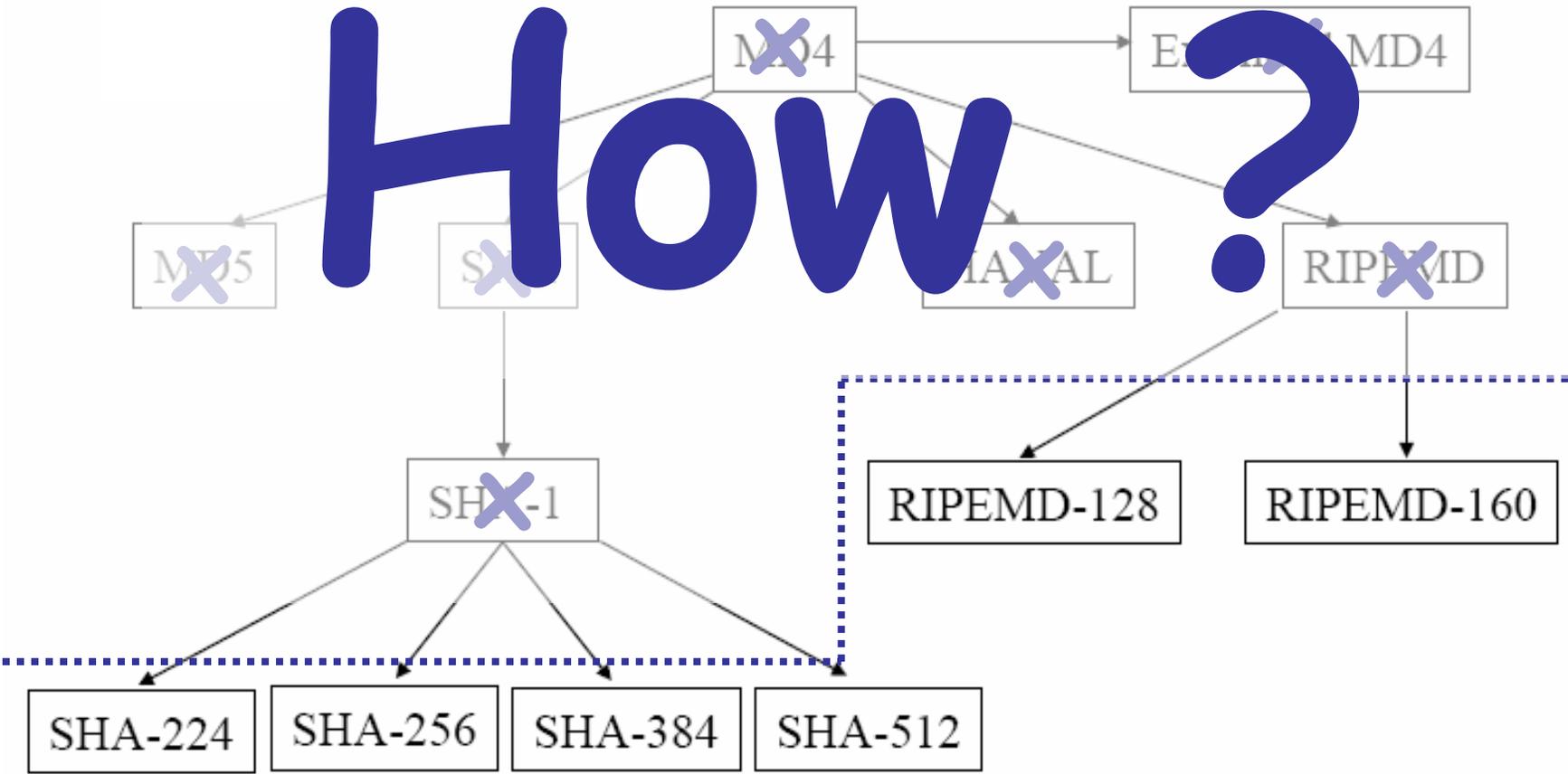


Motivation



Motivation

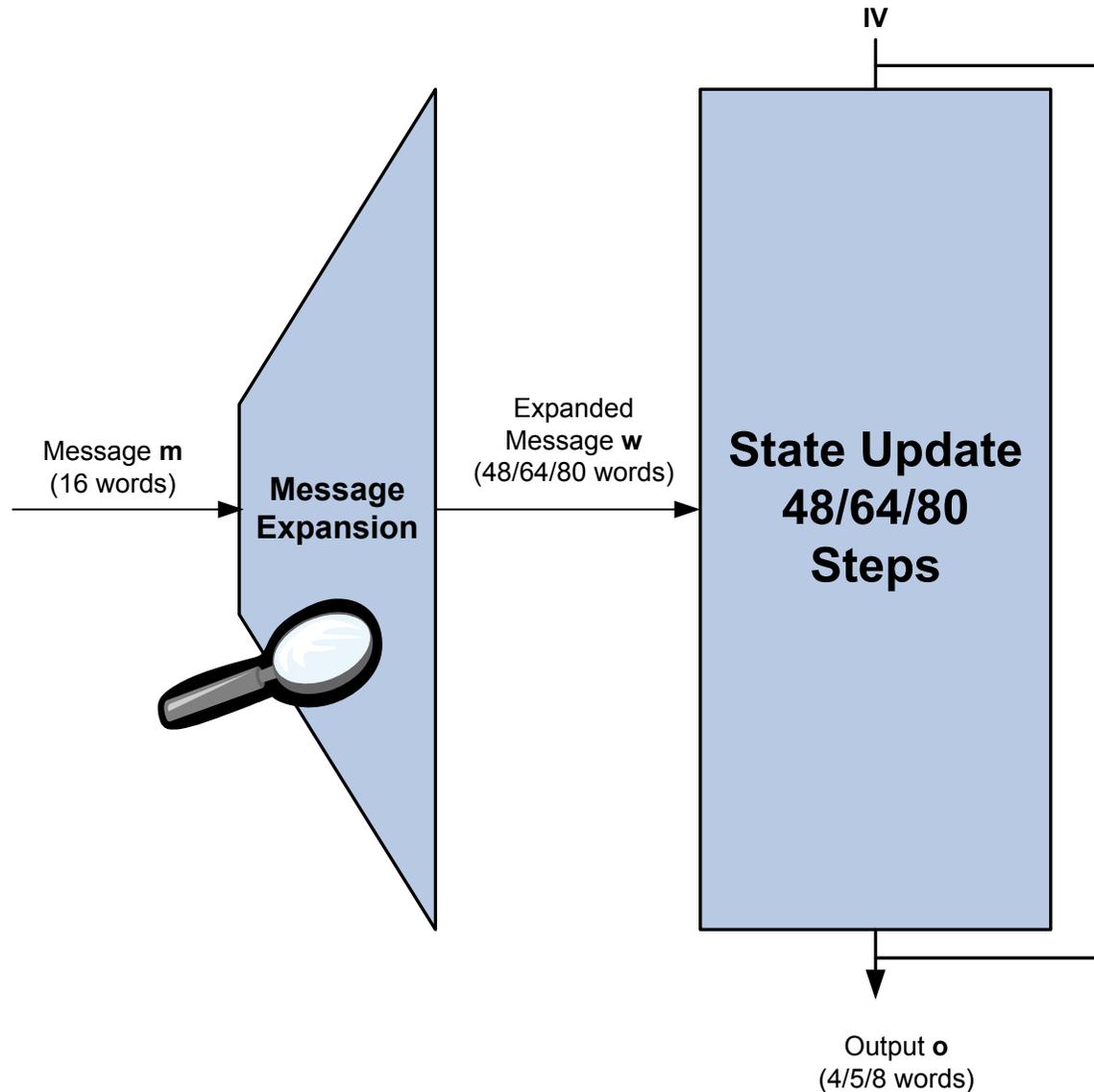
How?



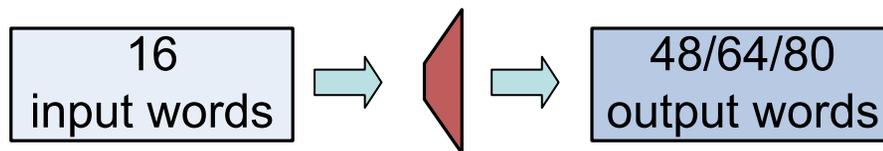
Agenda

- The MD4 “family”
 - Basic attack on SHA
 - Advanced methods for fast collision search (example SHA-1) and applications
 - SHA-2?
- Conclusions

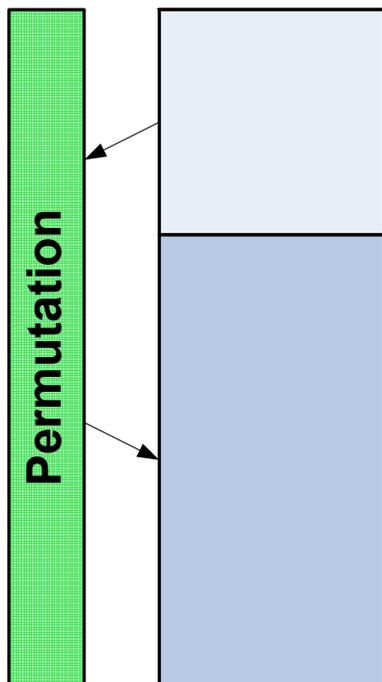
Outline of MD4-style Hash Functions



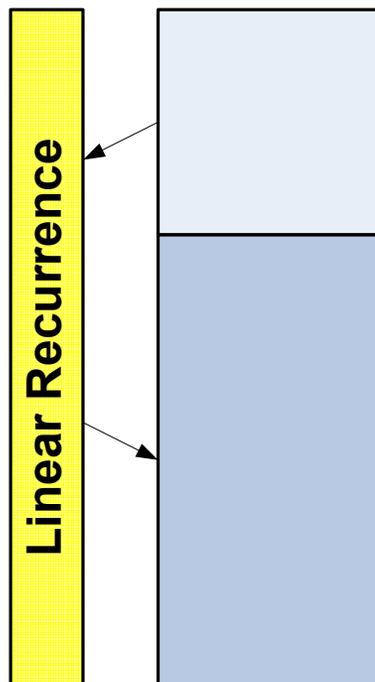
Message Expansions in the MD4 family



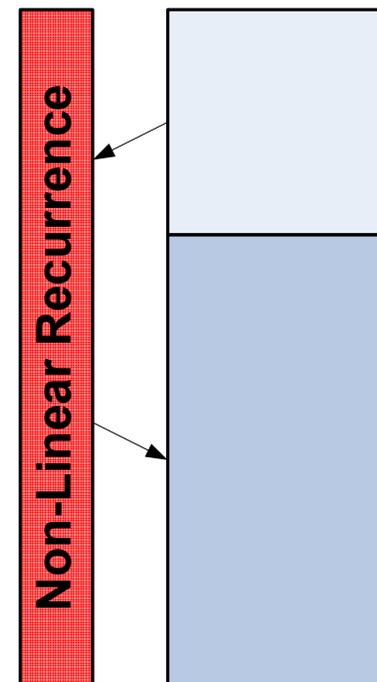
MD4/5, RIPEMD



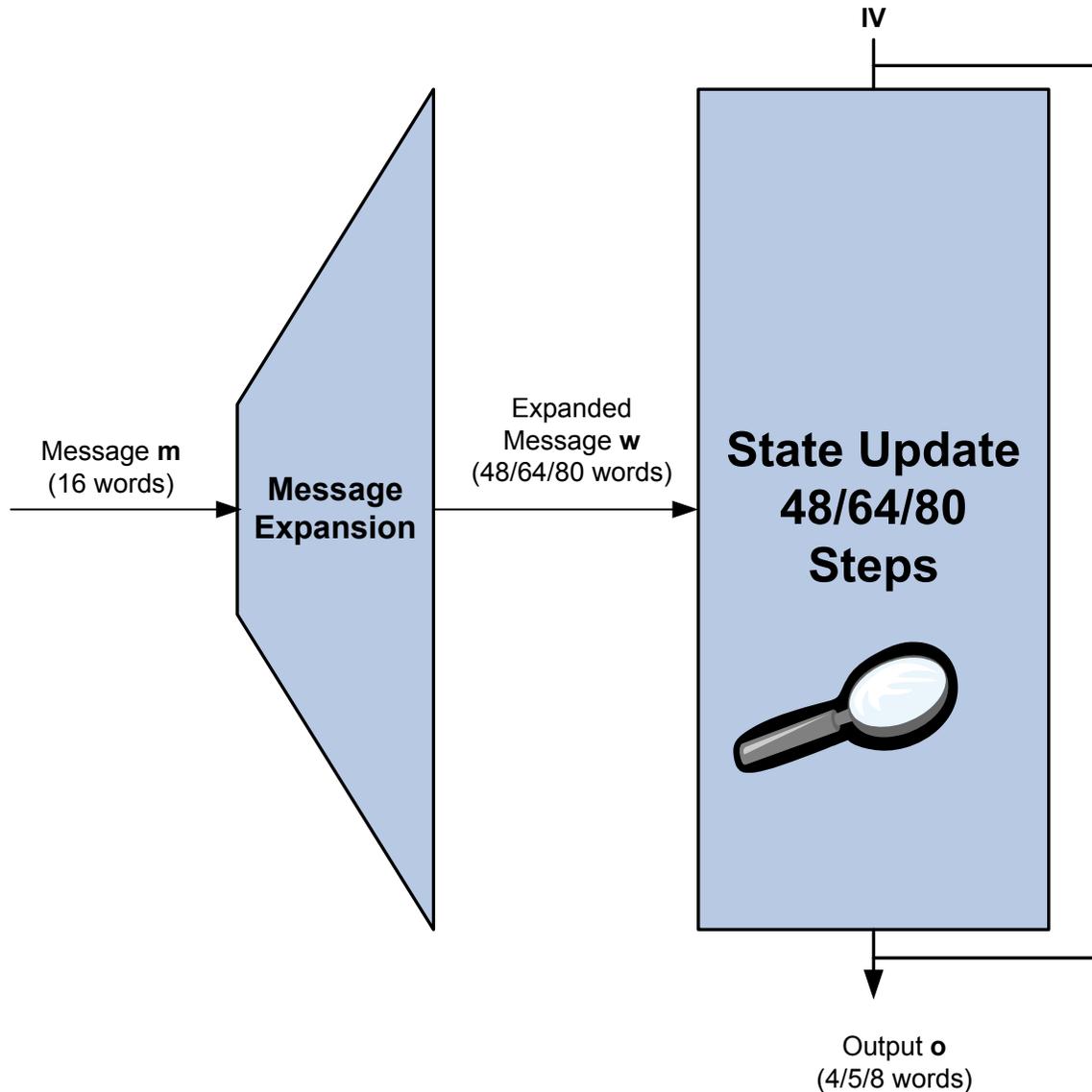
SHA / SHA-1



SHA-2 members

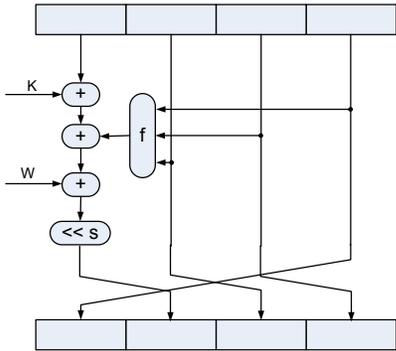


Outline of MD4-style Hash Functions

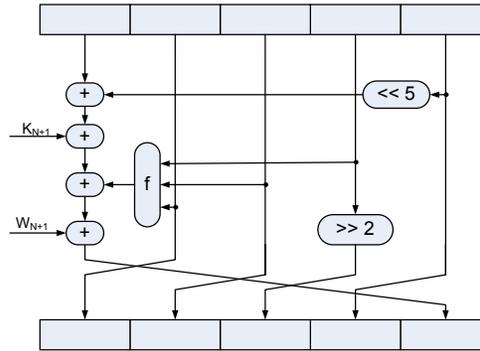


Evolution of the State Updates in the MD4 Family

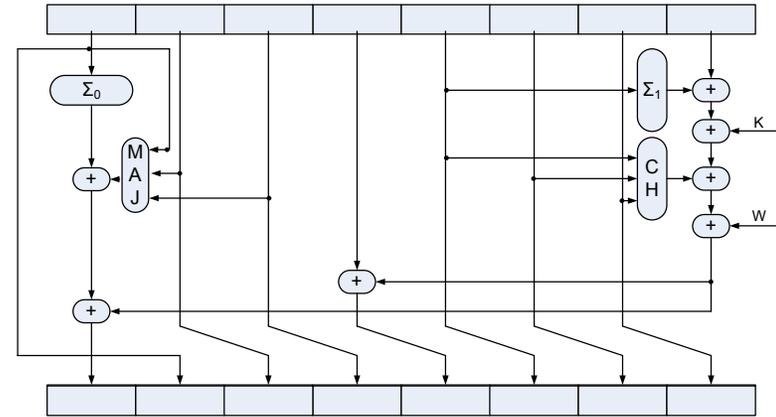
MD4



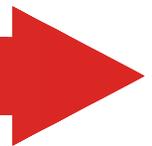
SHA/SHA-1



SHA-2 members



Design Complexity



Cryptanalysis of MD4

- Several people have shown weaknesses in the compression function of MD4
 - Merkle [Mer90]
 - Bosselaers and den Boer [BB91]
 - Vaudenay [Vau94]
 - Dobbertin [Dob96]

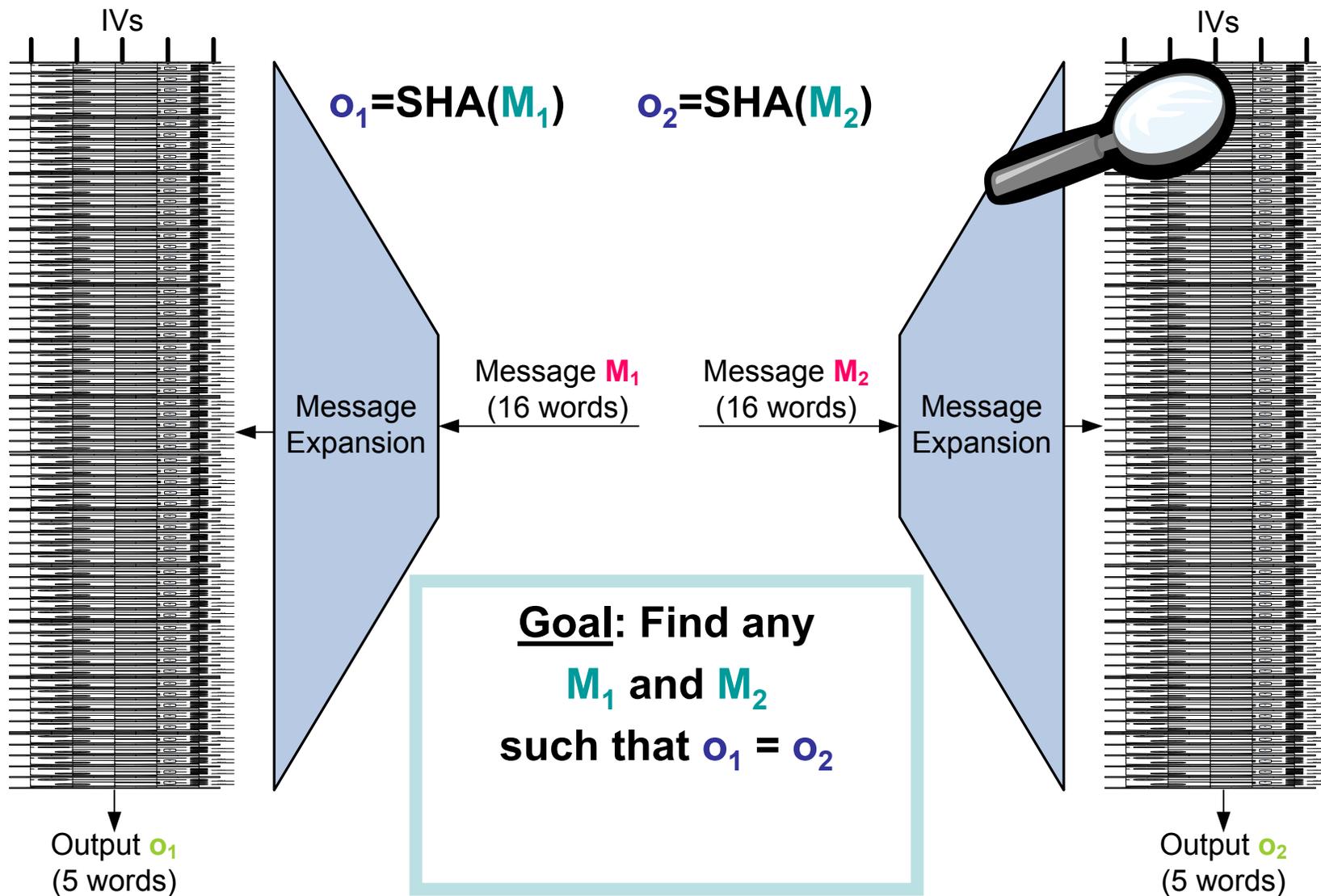
- Dobbertin's techniques received the most attention in the scientific community [Dob98]
 - Lead to a fast collision-producing algorithm
 - Even (partially) meaningful collisions can be produced

Cryptanalysis of SHA

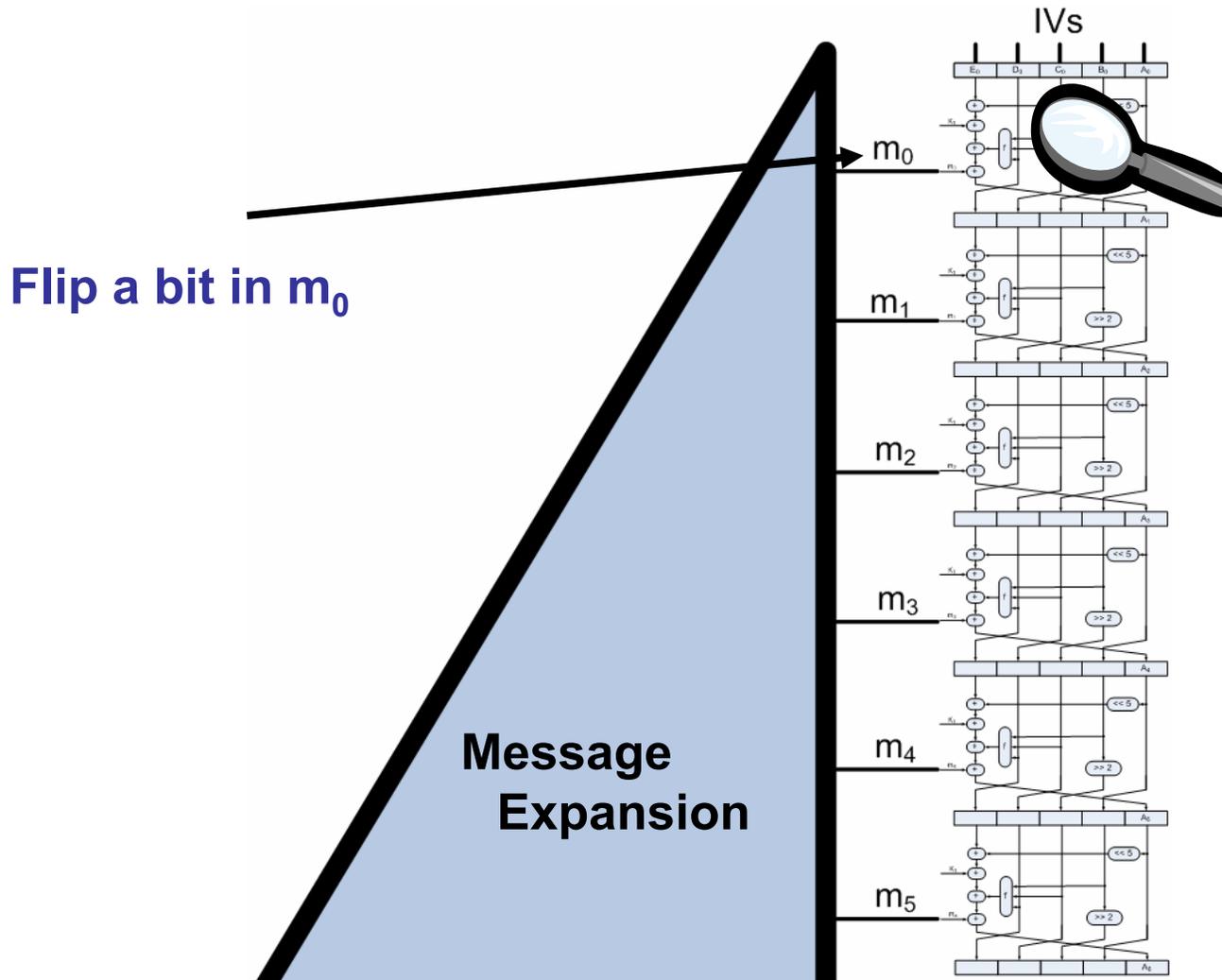
- SHA
 - Chabaud and Joux [CJ98]
 - Biham and Chen [BC04]
 - Joux et al. [BCJ+05]
 - Wang et al. [WYY05a]

- SHA-1
 - Rijmen and Oswald [RO05]
 - Biham and Chen [BCJ+05]
 - Wang et al. [WYY05b]
 - De Cannière and Rechberger [DR06]

How to produce a collision?

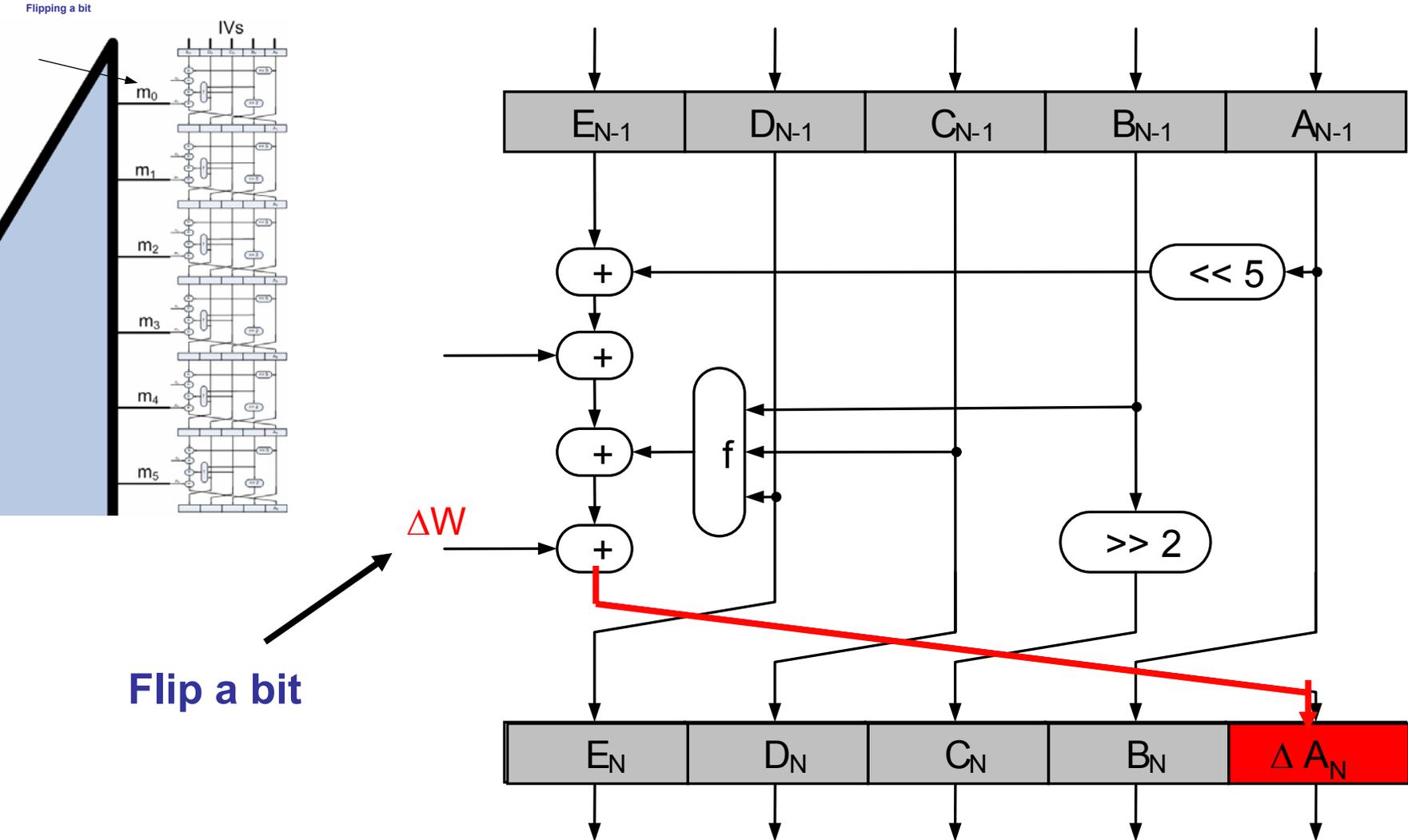


Propagation of a small difference



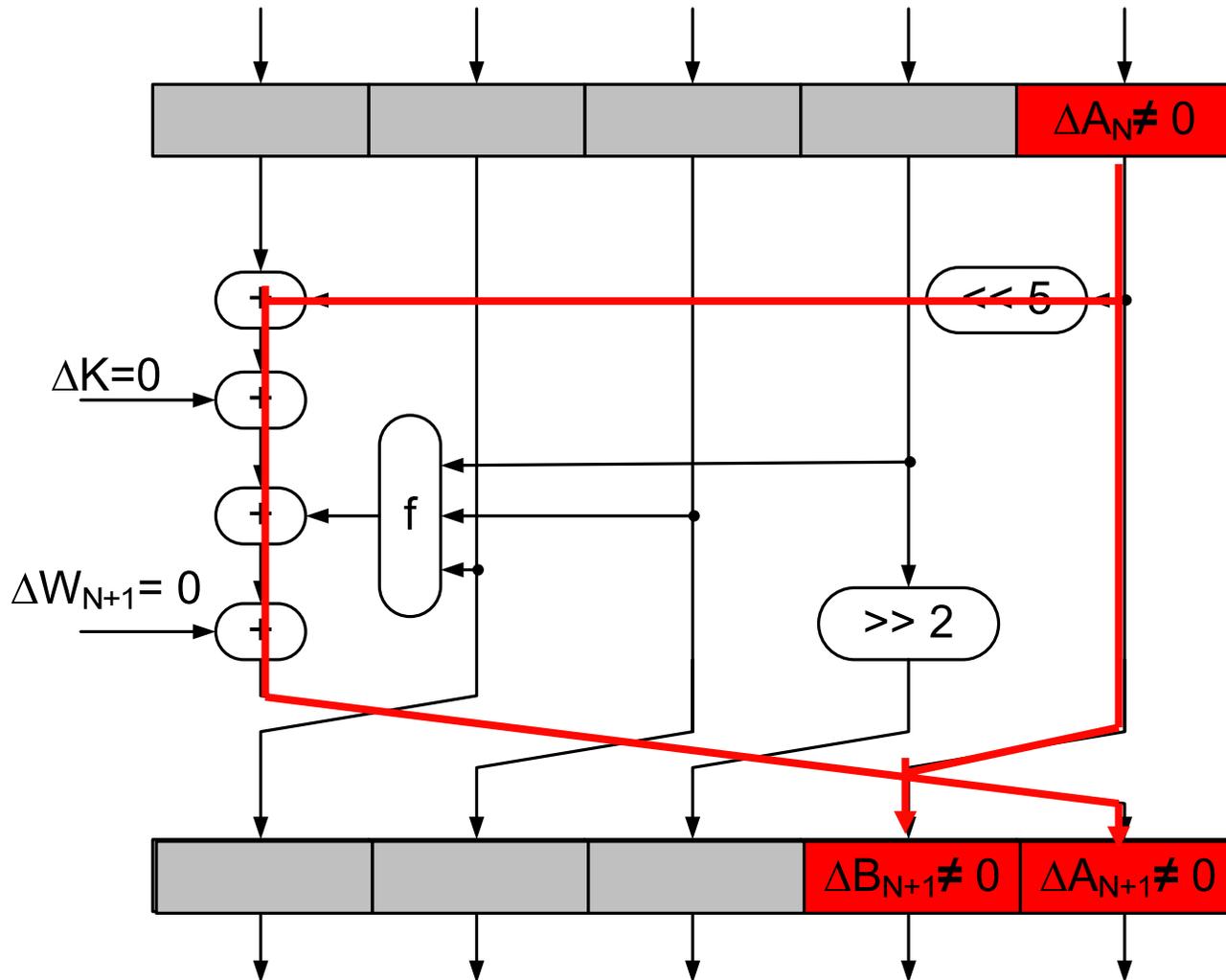
Propagation of a small difference

Step N



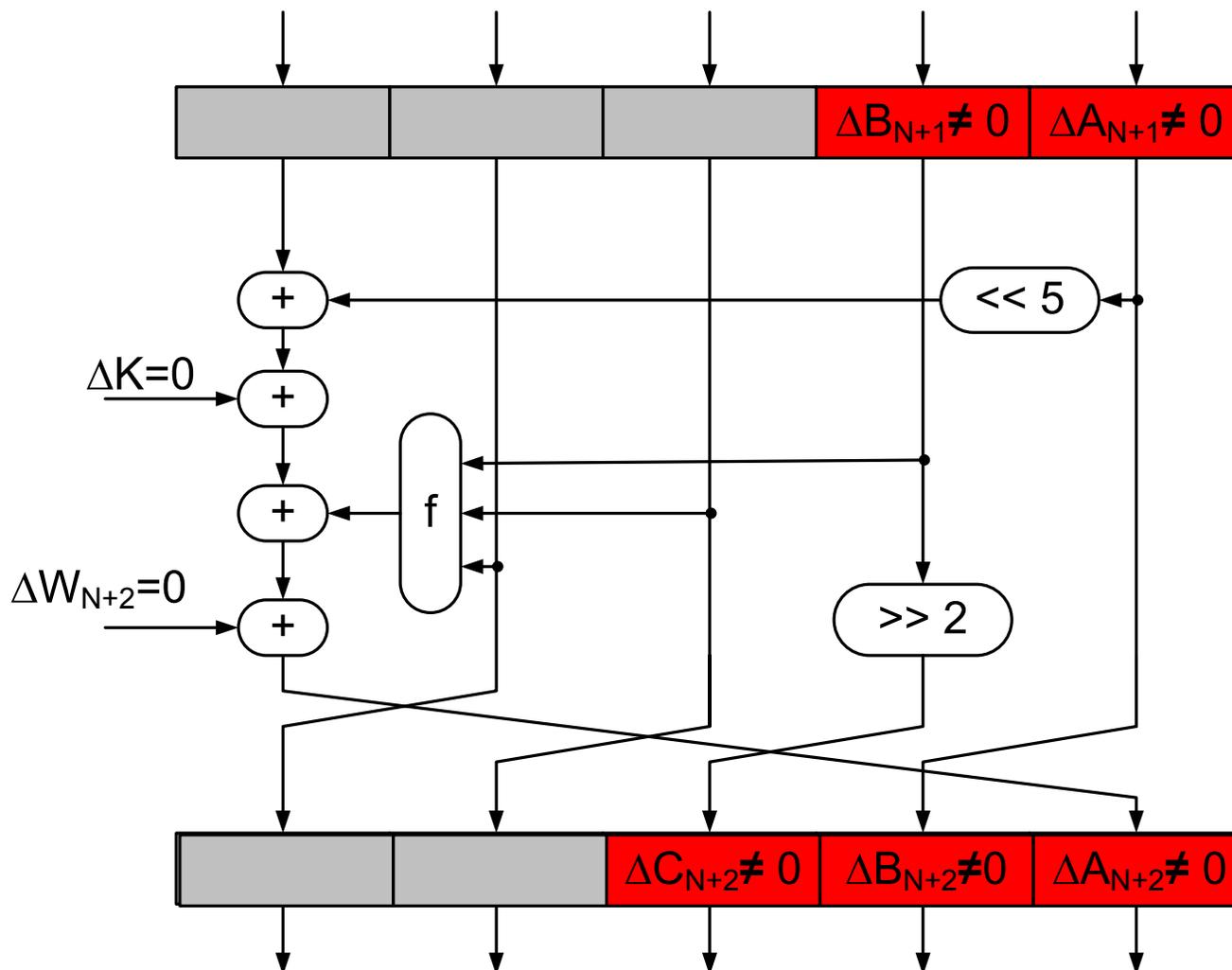
Propagation of a small difference

Step N+1



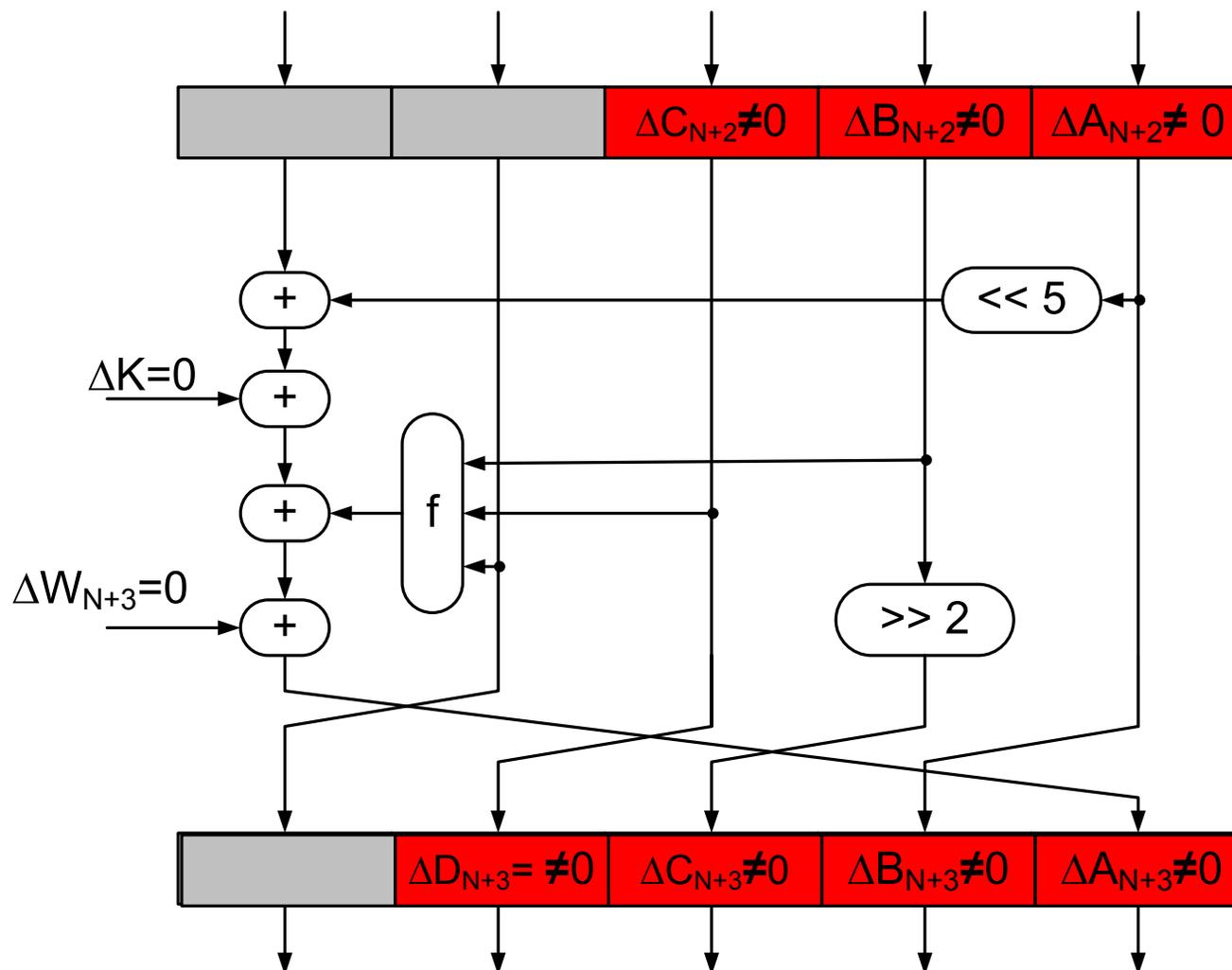
Propagation of a small difference

Step N+2



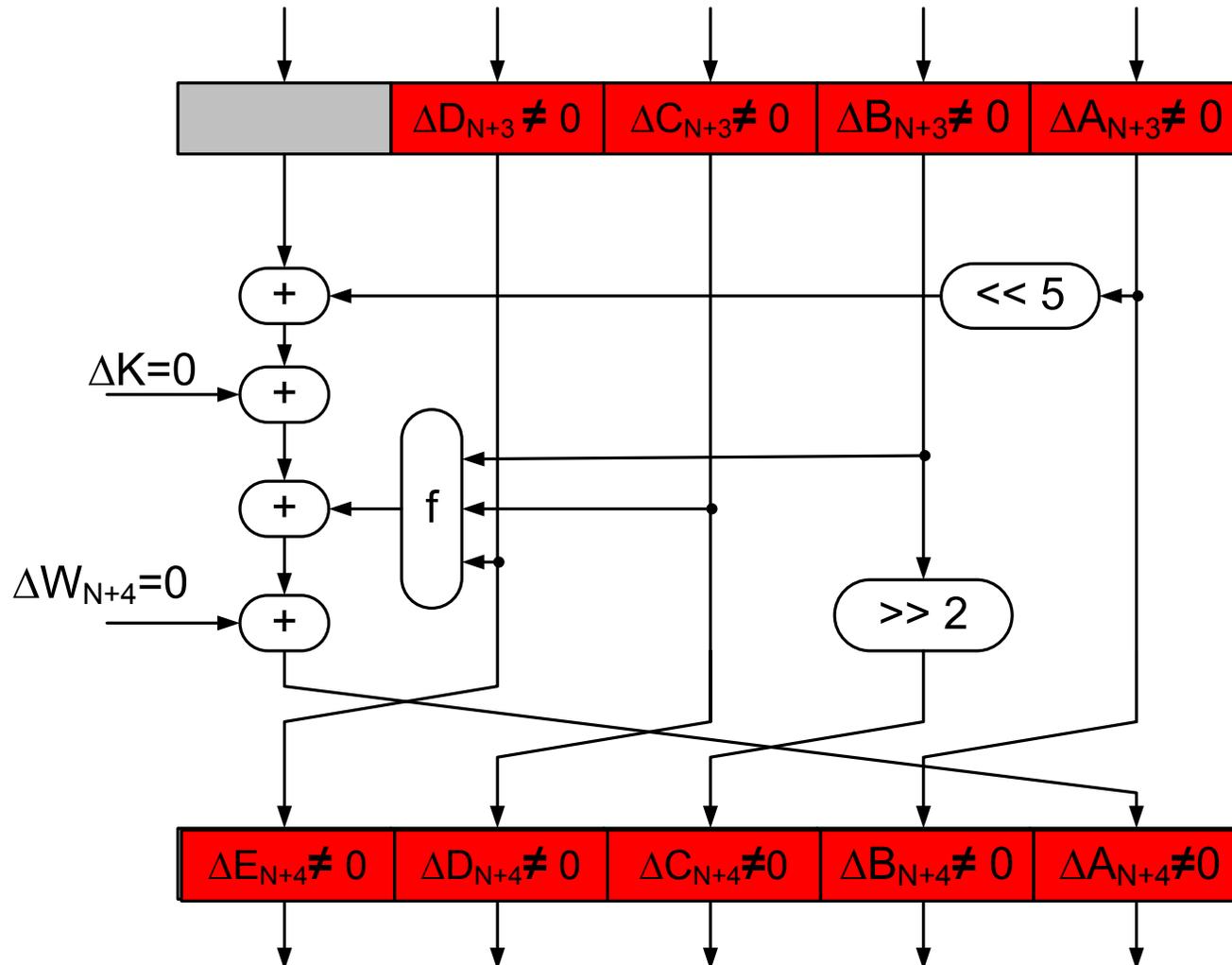
Propagation of a small difference

Step N+3



Propagation of a small difference

Step N+4



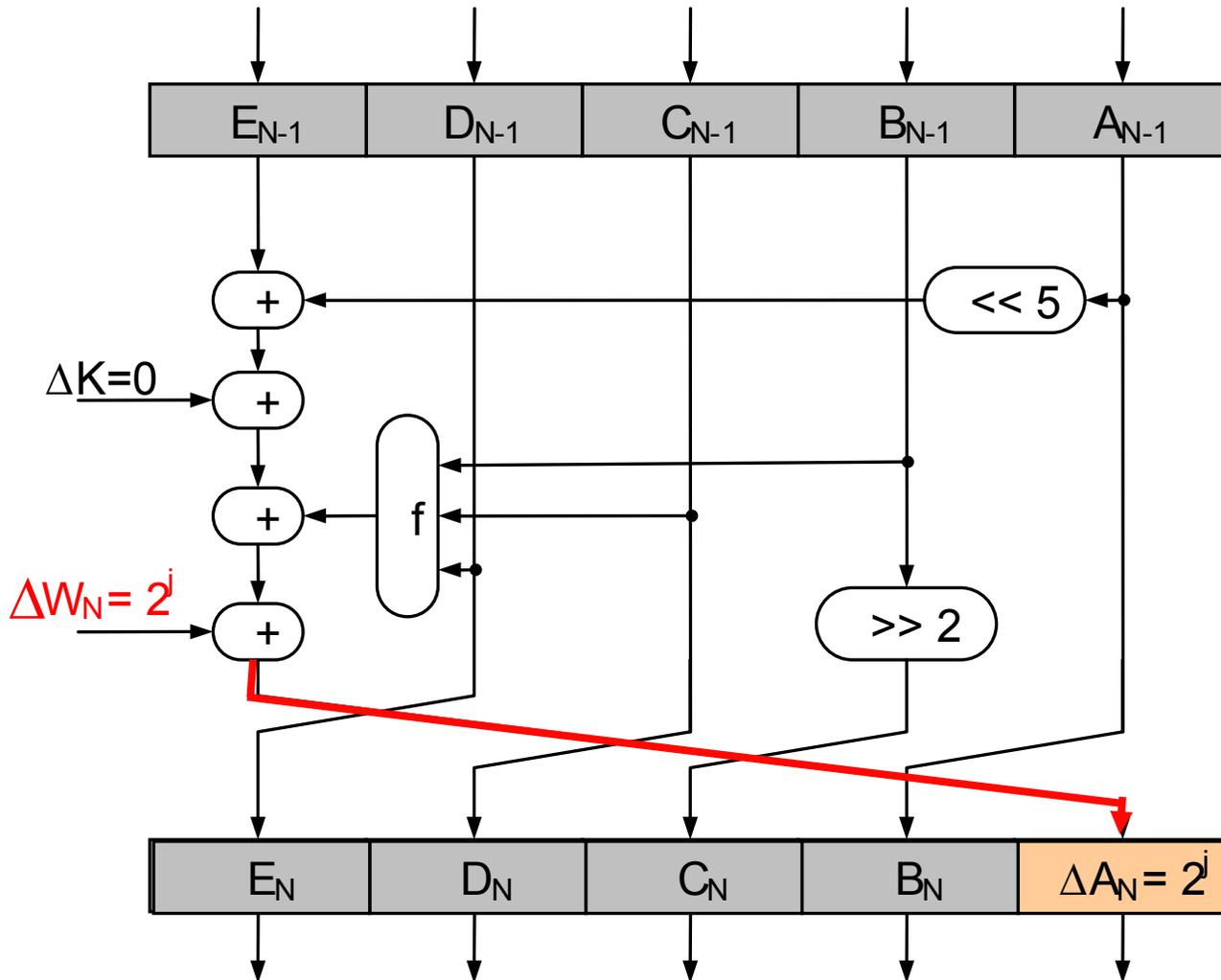
Differential cryptanalysis of SHA

- Small differences quickly expand
- Approach by Chabaud & Joux [CJ98]
 - Perturbations and corrections
 - Theoretical attack on SHA
 - Later on improved to practical attack [BCJ+05]

- Difference = XOR operation

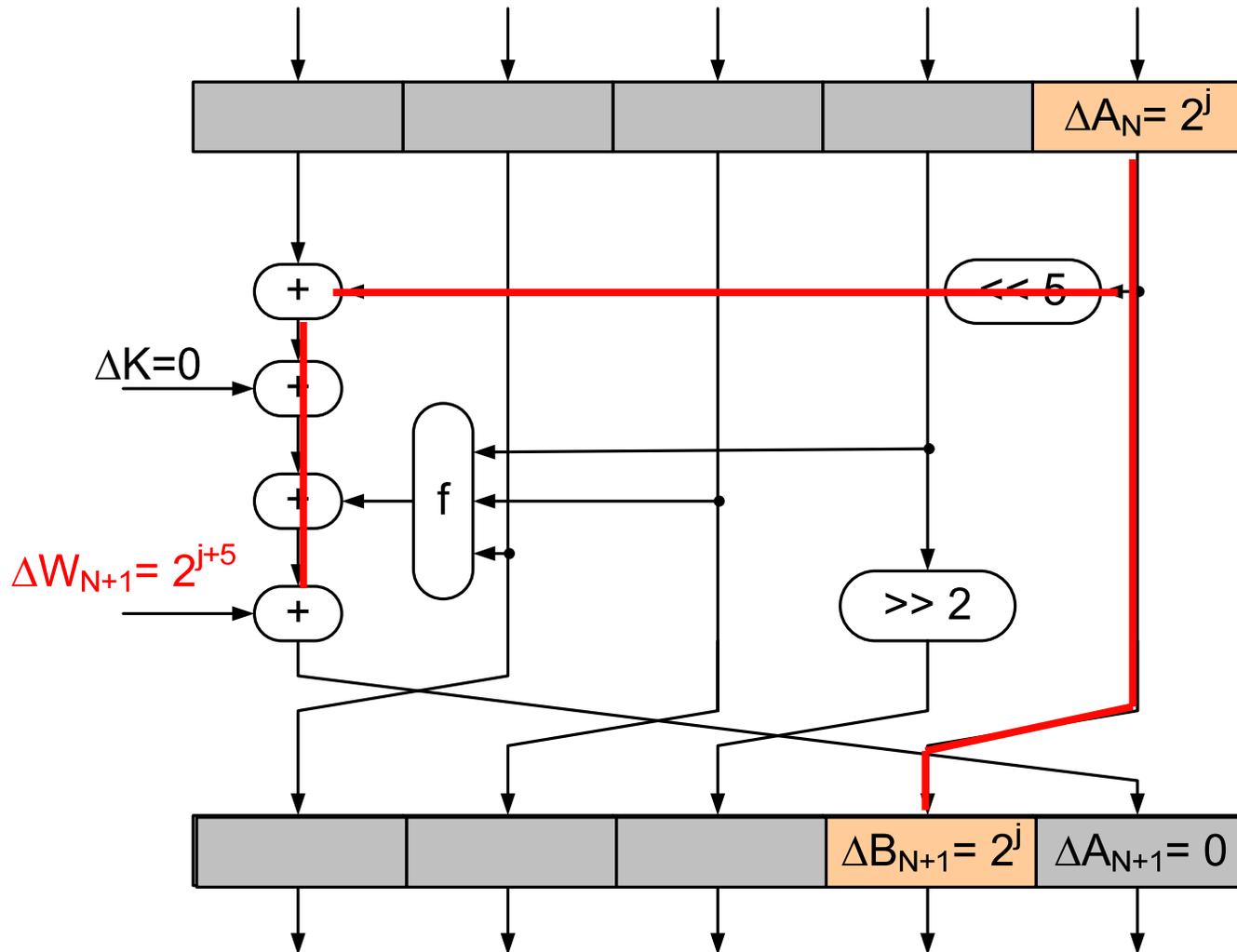
Perturbation

Step N



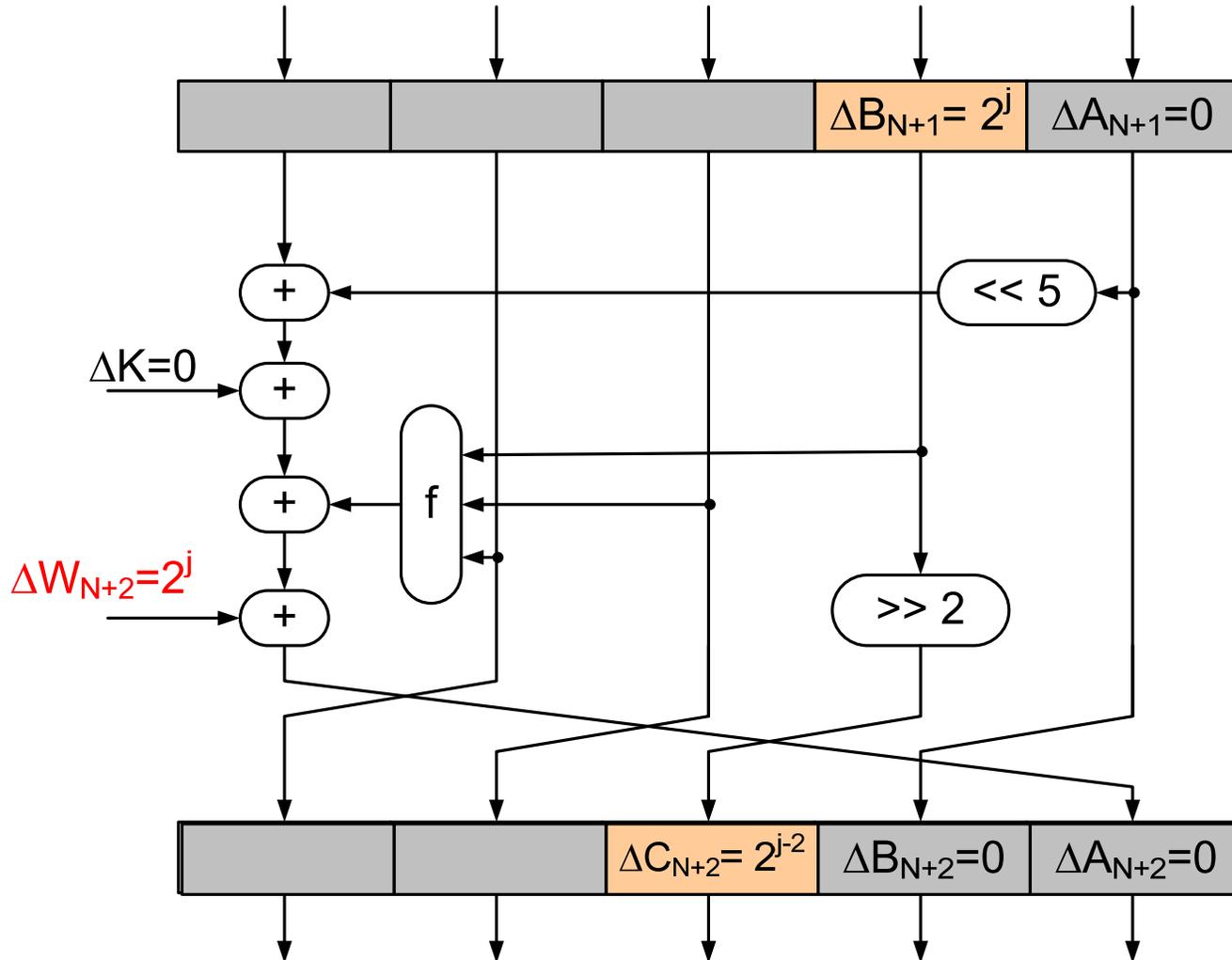
Correction 1

Step N+1



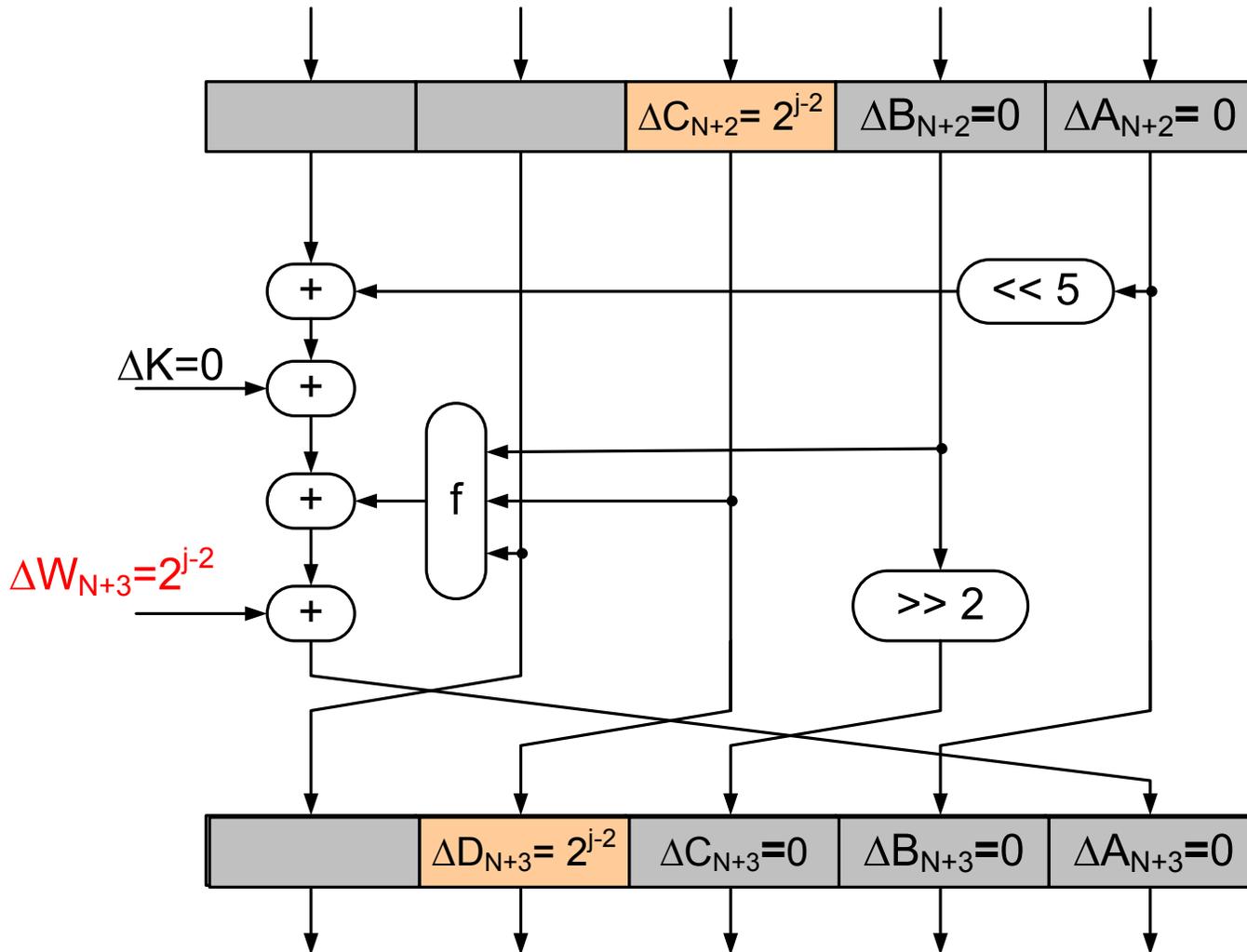
Correction 2

Step N+2



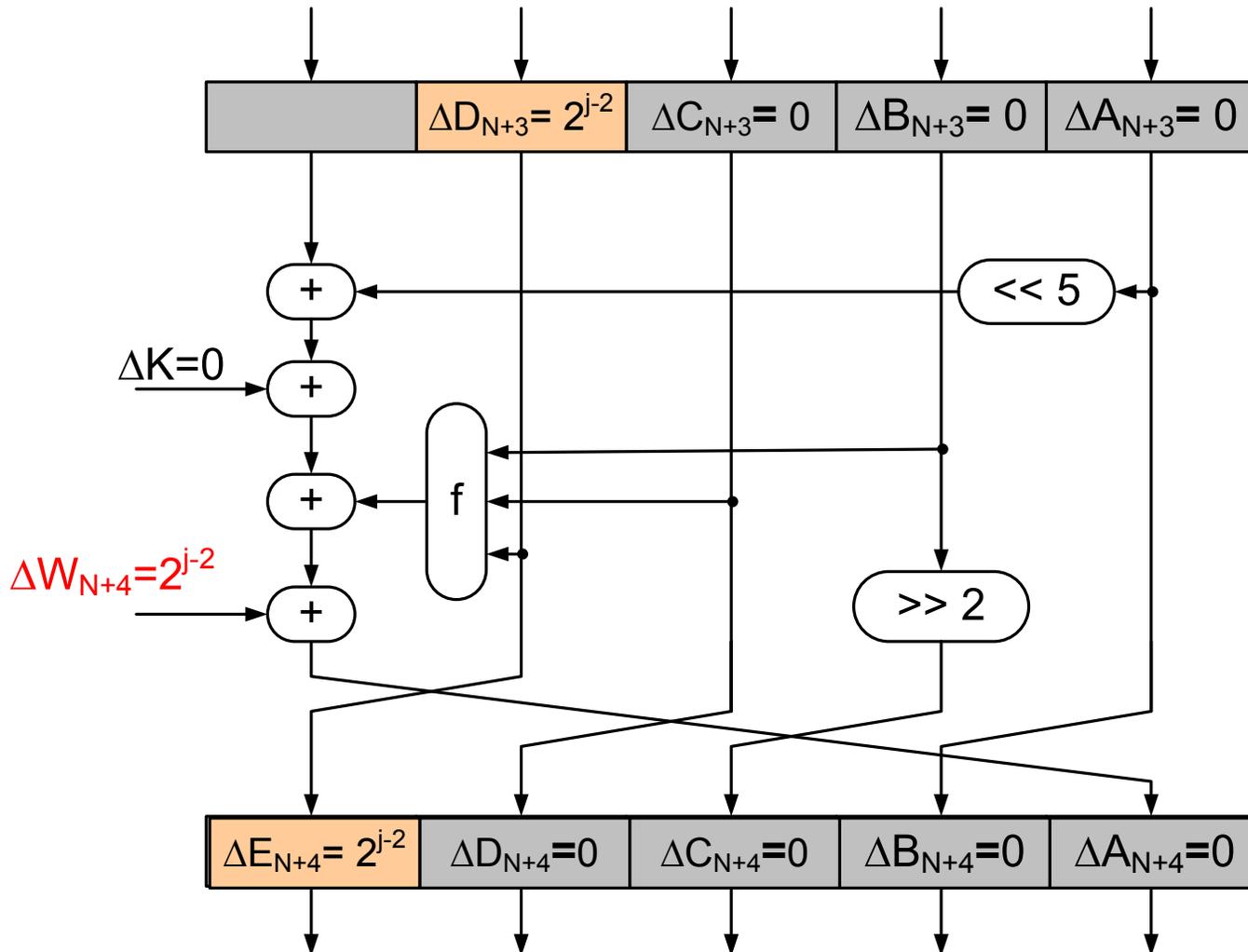
Correction 3

Step N+3



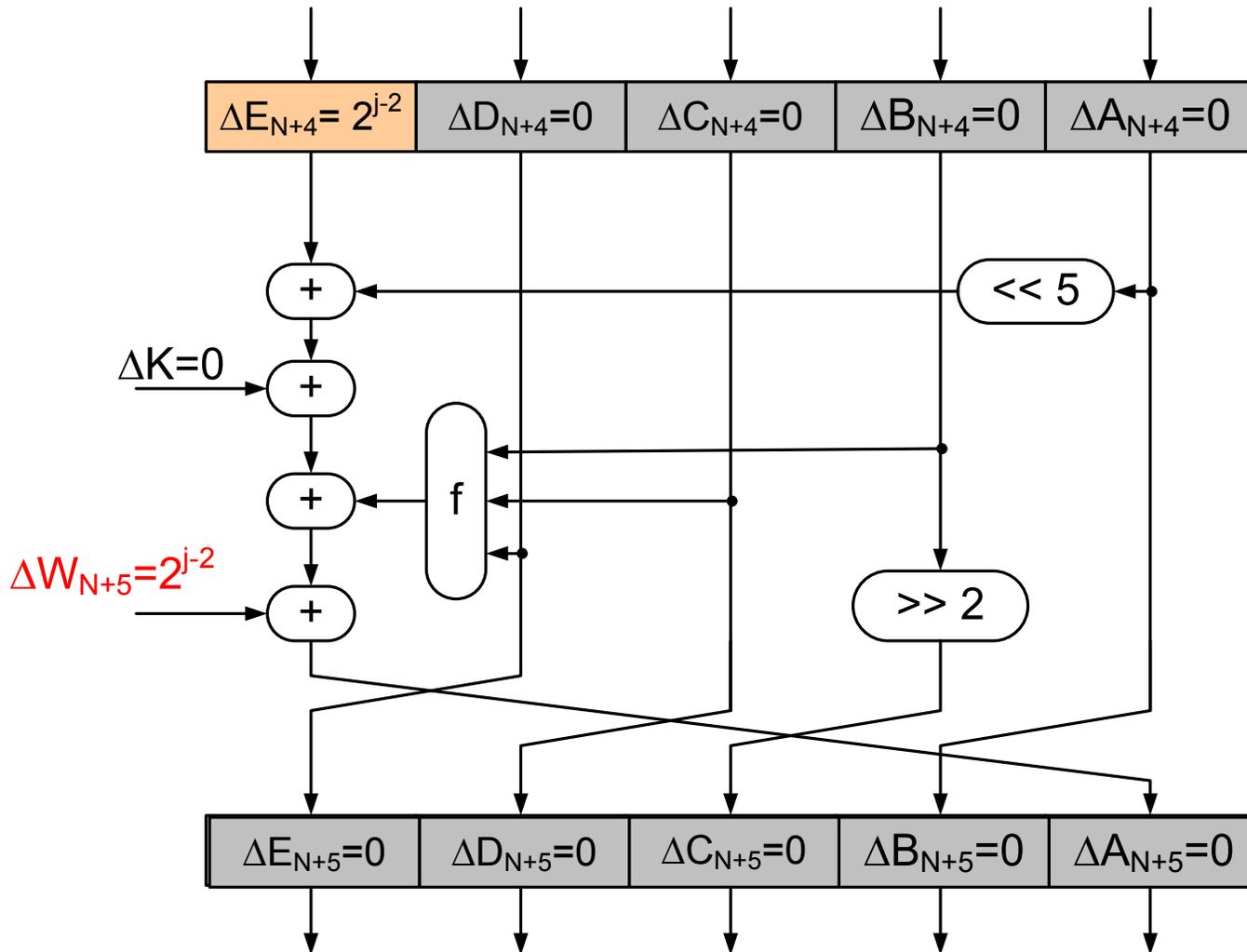
Correction 4

Step N+4



Correction 5

Step N+5



Local collision

- Resynchronisation of internal state
- One perturbation and 2-5 corrections
- Creating local collisions is not so difficult
- Problem: message expansion

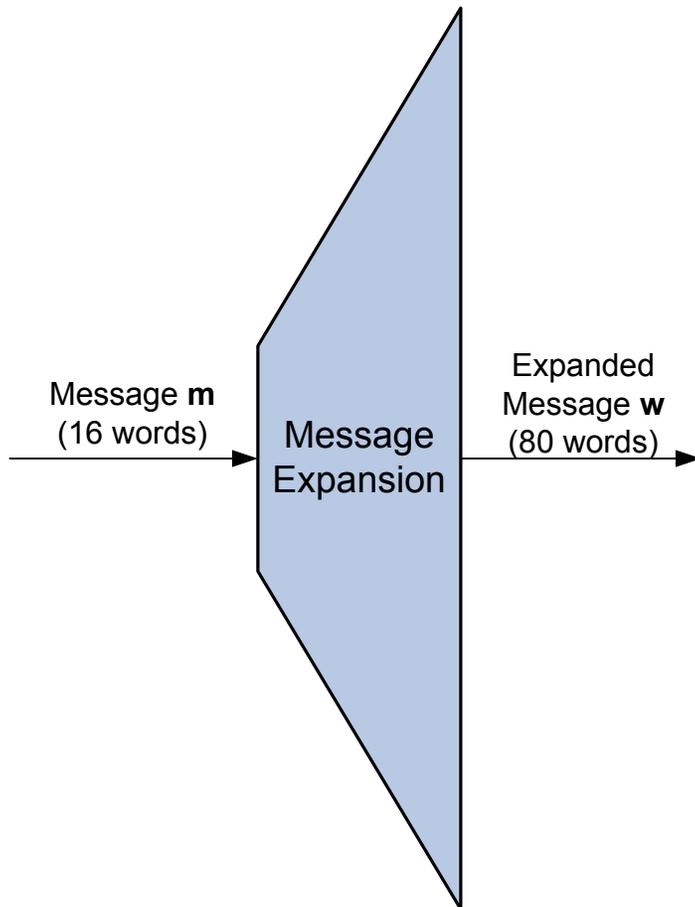
Outline of SHA – Message Expansion

SHA

$$W_t = \begin{cases} M_t & \text{for } (0 \leq t \leq 15) \\ W_{t-3} \oplus W_{t-8} \oplus W_{t-14} \oplus W_{t-16} & \text{for } (16 \leq t \leq 79) \end{cases}$$

SHA-1

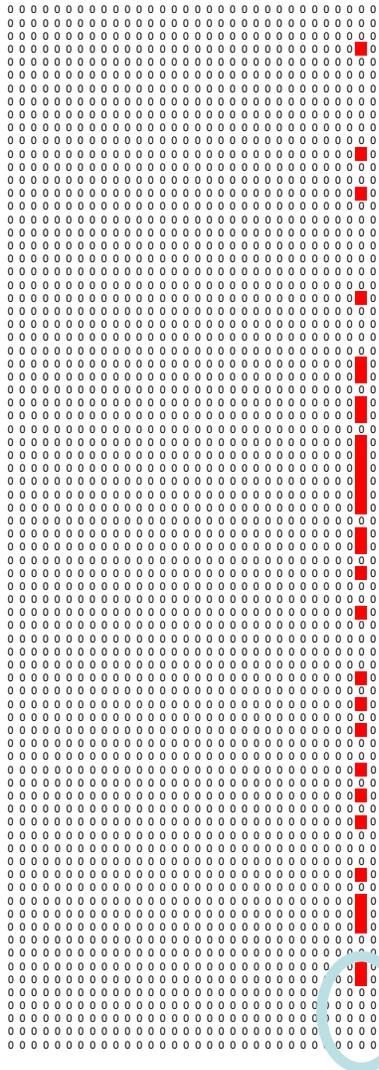
$$W_t = \begin{cases} M_t & \text{for } (0 \leq t \leq 15) \\ (W_{t-3} \oplus W_{t-8} \oplus W_{t-14} \oplus W_{t-16}) \lll 1 & \text{for } (16 \leq t \leq 79) \end{cases}$$



Message expansion

- Every bit changed influences other bits
- Impossible to find m_1, m_2 such that $ME(m_1), ME(m_2)$ differ in 3-6 bits only
- Constructing a global collision = finding good characteristic

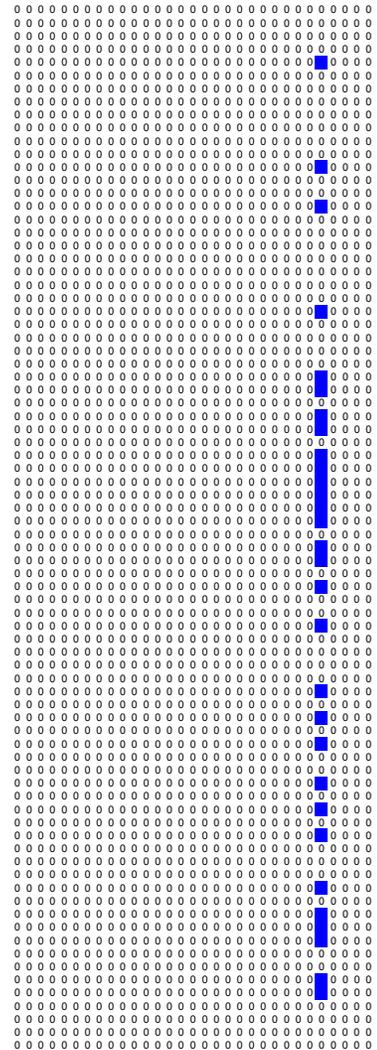
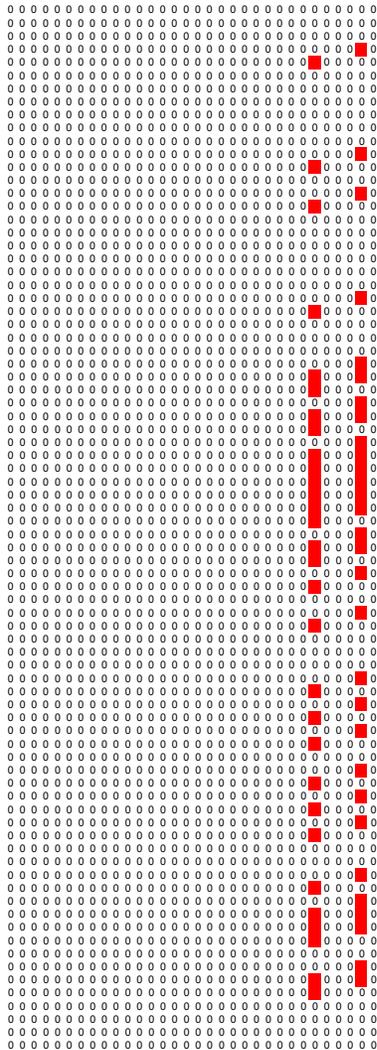
Building a collision for SHA



- Perturbation pattern
- Low weight

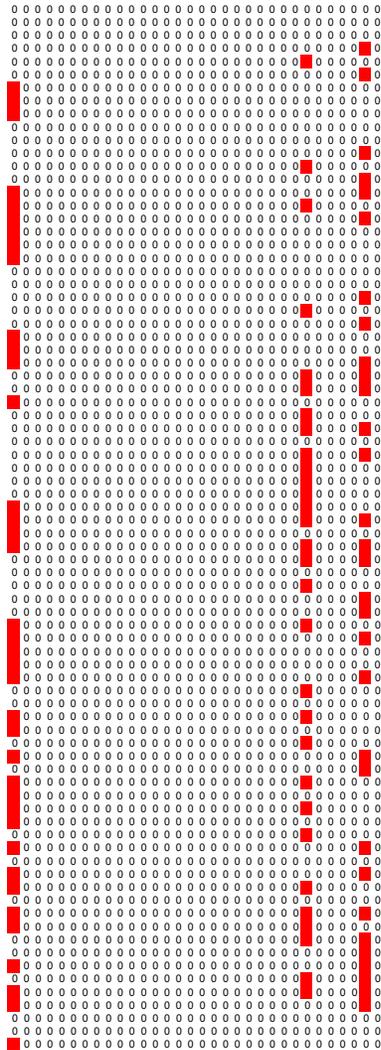
$$W_t = W_{t-3} \oplus W_{t-8} \oplus W_{t-14} \oplus W_{t-16}$$

A collision-producing difference pattern



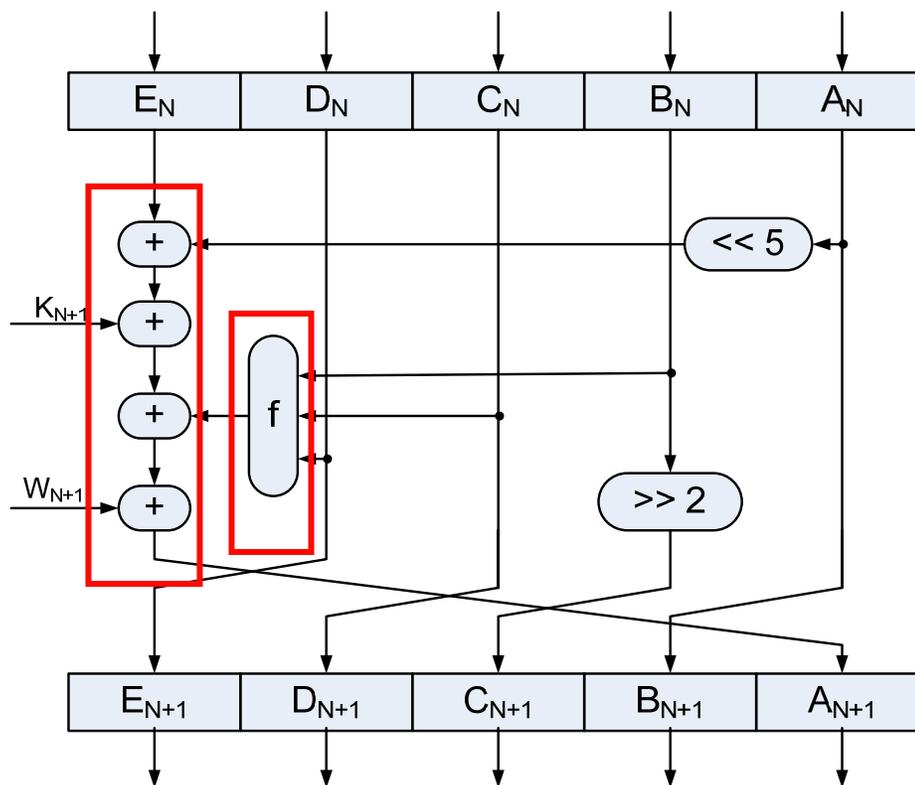
- Apply 5 corrections with the same pattern
 - displaced over steps
 - rotated over bit positions

A collision-producing expanded-message difference pattern



- Completed difference pattern consisting of
 - 1 perturbation pattern
 - 5 correction patterns

Conditions imposed by nonlinear elements



- Boolean function f
- Modular addition

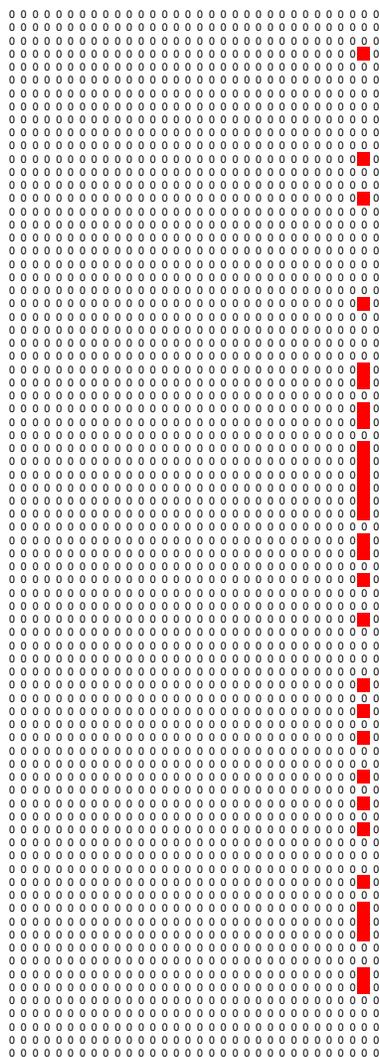
Modular addition

- Linear except for carry effects
 - Carry = 0 with probability 1/2
 - Carry moves upwards only
 - No carry from MSB
- Requirement: difference propagation as with XOR

Boolean functions

- Linear in 40 out of 80 steps
- Bitwise parallel: every input bit affects 1 output bit
- We set as requirement: difference propagations as with XOR
 - High probability
 - Easy to find good characteristics

Conditions

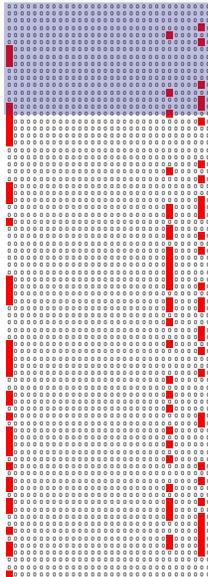


- Every perturbation gives 2-5 conditions on the message
- Most conditions are nonlinear and complicated to express in terms of the input message
- Goal is to minimize these conditions (to make final search easier)

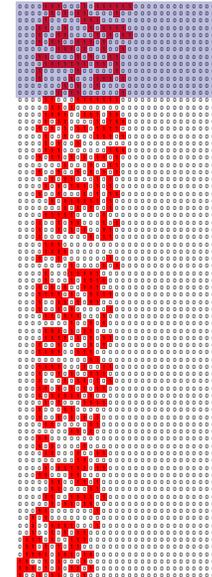
Results of CJ98

- Low- weight patterns exist for SHA => break

SHA



SHA-1



- For SHA-1: weight is too high





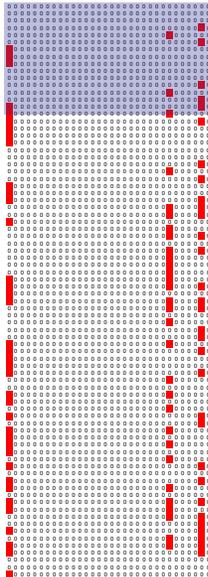




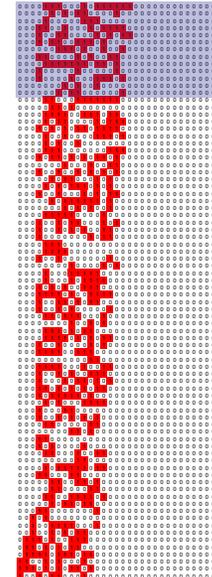
Results of CJ98

- Low- weight patterns exist for SHA => break

SHA



SHA-1



- For SHA-1: weight is too high

Improvements

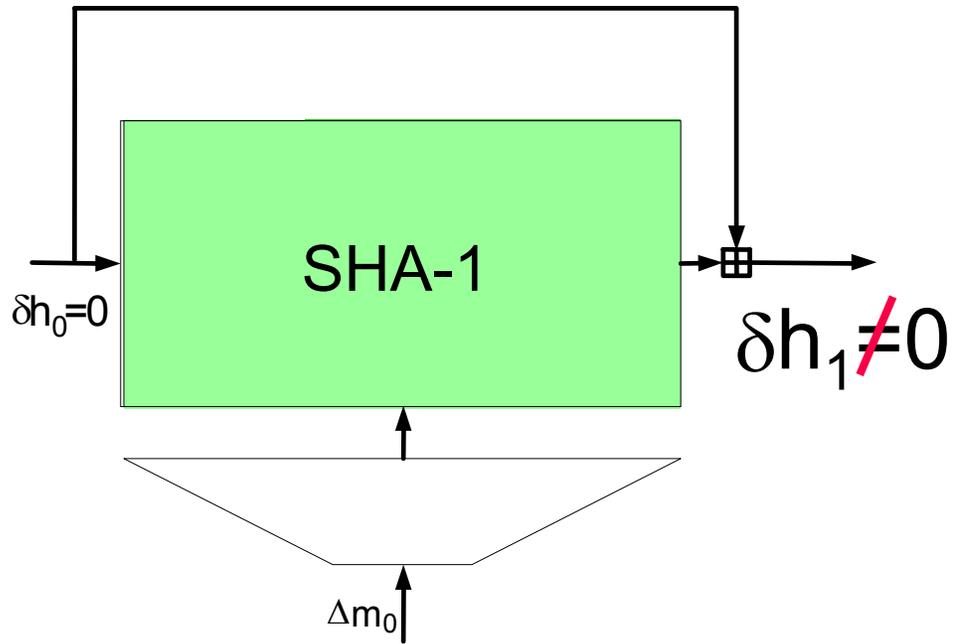
- Better characteristics
 - 1-block → multi-block
 - Better suited for hash functions

- Better ways to construct right pairs
 - Message modification

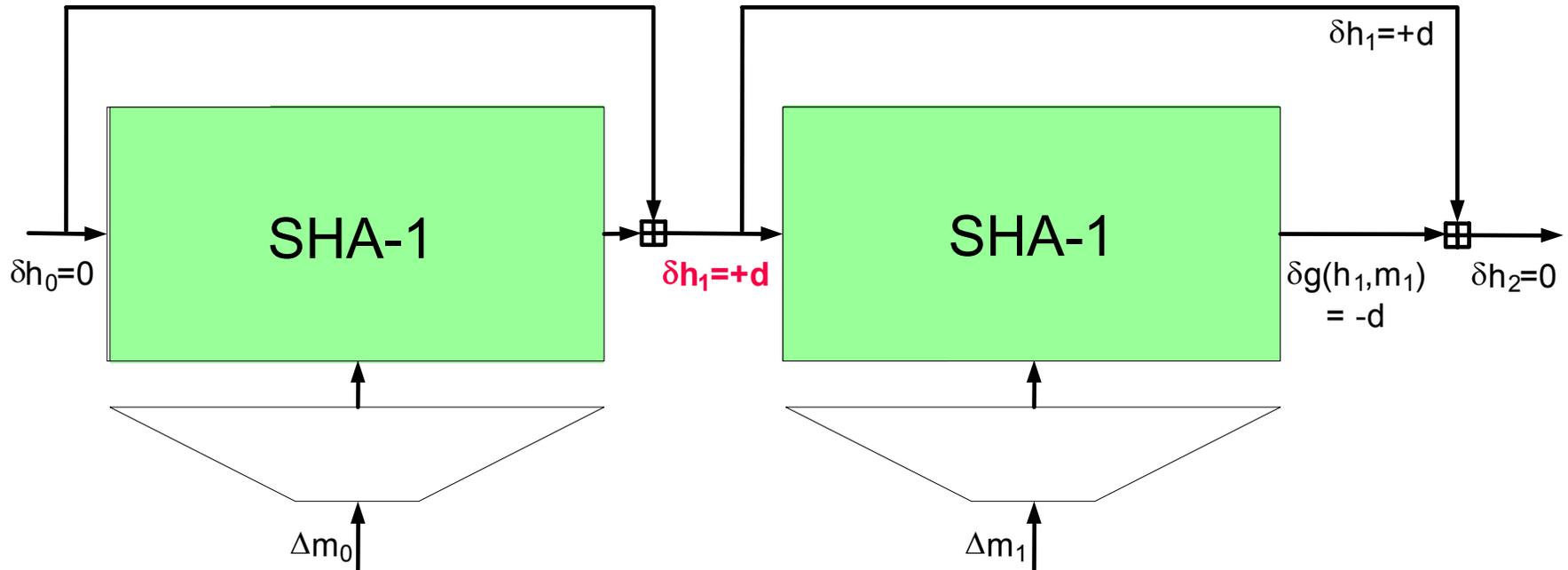
Multi-block collisions

- Near-collision: outputs differ in only a few bits
- Observation: much easier than collisions
 - Characteristics with higher probability

Use of near-collisions



Use of near-collisions

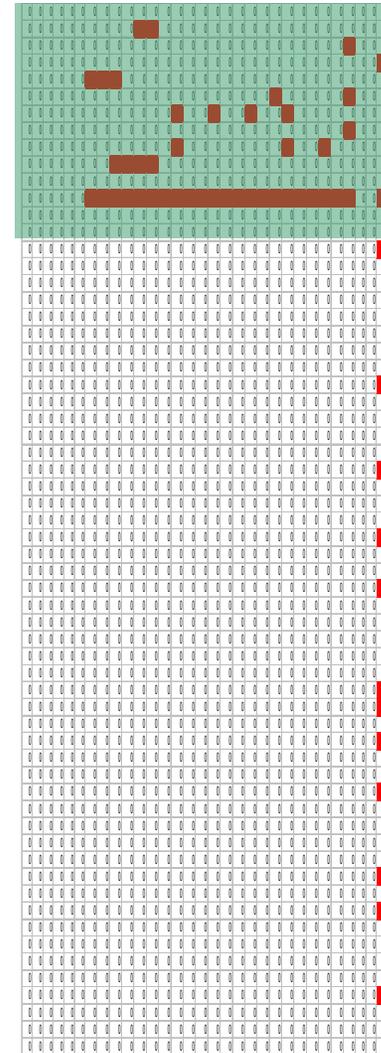
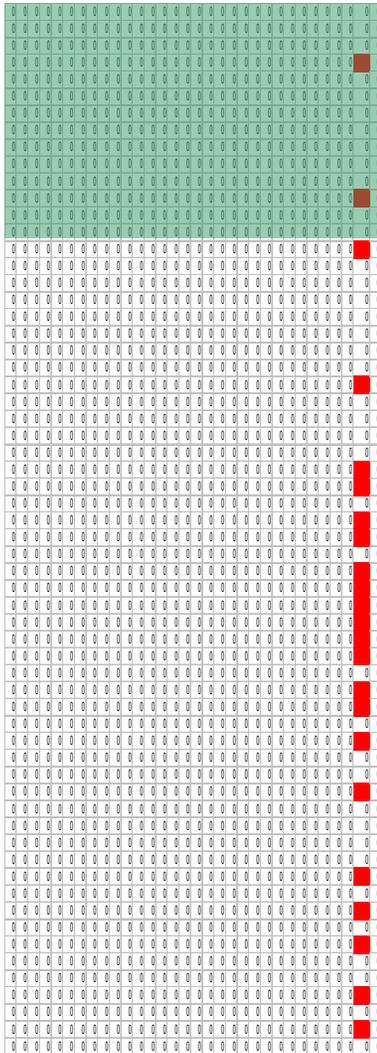


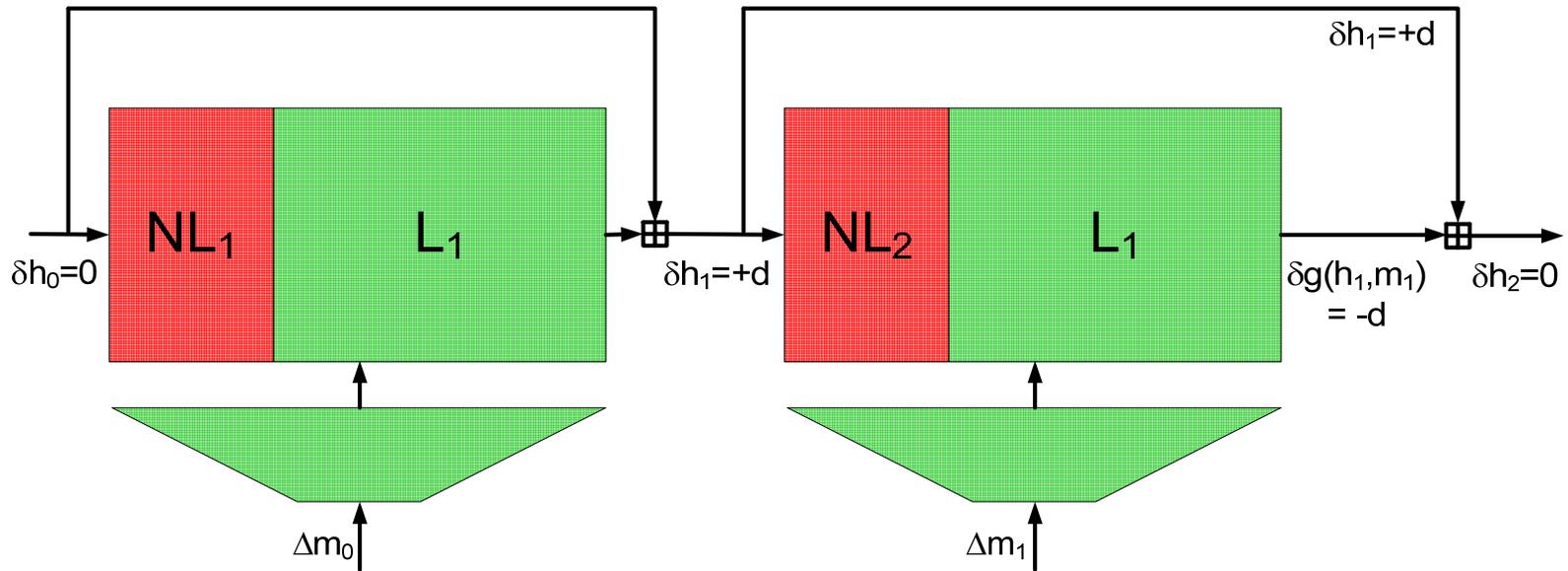
- Two related near-collisions give a 2-block collision
- Work effort of two blocks is only double of one block

Differential attacks on hash functions

- Good characteristics for block ciphers:
 - Optimise probability
 - Minimise number of chosen plaintexts
- Good characteristics for hash functions
 - Optimise probability
 - Minimise effort to solve equations
 - Equations in first steps are always easy
 - Only a small part of the message involved
 - Inputs are known
 - Late start / Early stop

Good characteristics





- Two key techniques of Wang et al.:
 - Manually find suitable complex characteristic NL_1 and NL_2
 - Advanced message modification to improve work factor
- Methods are rather ad hoc (manual)
- Optimization?

Optimising characteristic

- Optimise probability after step 16
- Before step 16
 - Concentrate low probability in few steps
 - More difficult search problem
 - Manual construction [WYY05b]
 - Automatic tools [DR06]

How does it work?

Principles

- Generalized conditions

| x_i | x_i^* |
|-------|---------|
| 0 | 0 |
| 0 | 1 |
| 1 | 0 |
| 1 | 1 |

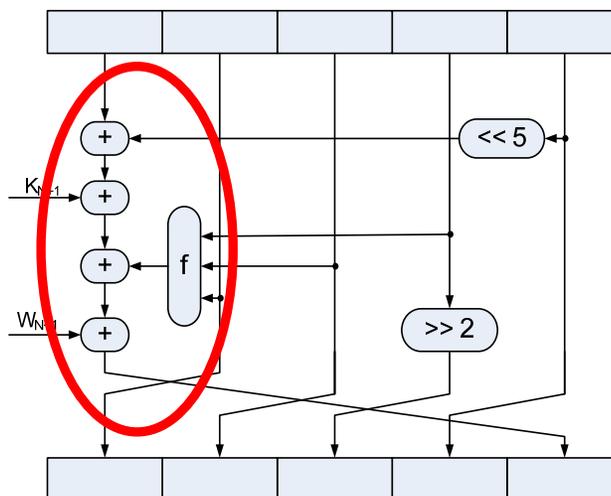
| Type | Possibilities |
|---------------------|---------------|
| XOR | 2 |
| Signed-bit | 4-6 |
| Generalized: | 16 |

Generalized Conditions - Notation

| (x_i, x_i^*) | $(0, 0)$ | $(1, 0)$ | $(0, 1)$ | $(1, 1)$ |
|----------------|----------|----------|----------|----------|
| ? | ✓ | ✓ | ✓ | ✓ |
| - | ✓ | - | - | ✓ |
| x | - | ✓ | ✓ | - |
| 0 | ✓ | - | - | - |
| u | - | ✓ | - | - |
| n | - | - | ✓ | - |
| 1 | - | - | - | ✓ |
| # | - | - | - | - |

Principles

- Generalized conditions
- Use “bit-sliced design” to efficiently
 - Propagate conditions *within one* step transformation
 - Propagate conditions *among all* step transformations



Key properties of new approach [DR06]

- Looks for (parts of) the colliding pair and characteristic at the same time
- *Precise* calculation of probabilities instead of approx. by HW and counting conditions
- Available degrees of freedom can be used to
 - Direct improvements of probability (Greedy approach)
 - Facilitate final search method for a right pair
 - Fix part of the colliding message on beforehand and/or during final search (meaningful)

Constructing right pairs

- Equations following from nonlinear operations
 - Every step increases the complexity
- First 15 steps: easily solvable
- Next steps: mostly guess and verify
- Solving of eqs. in steps 16 and ff.
 - Neutral-bit technique [Biham and Chen]
 - Advanced message modification [Wang et al.]
 - Symbolic computation [Sugita et al.]
 - Greedy method [De Cannière and Rechberger]
 - Boomerang method [Joux and Peyrin]

Neutral bits

- Weak diffusion
- After small number of steps, not all output bits depend on all input bits
- When trying pairs, only vary bits that don't change the outputs which are already right

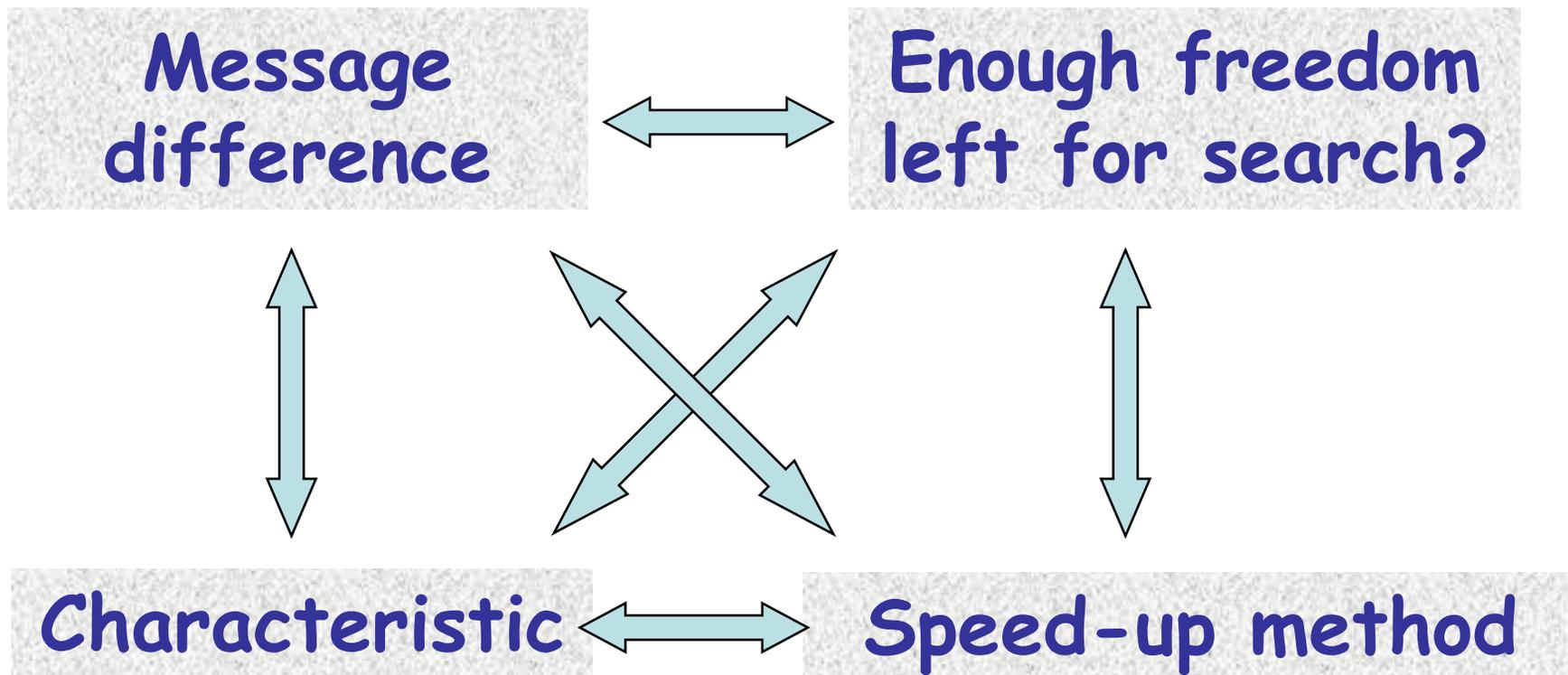
Advanced message modification

- Solve equations deterministically
 - Iterative solving strategy
- Again helped by weak diffusion

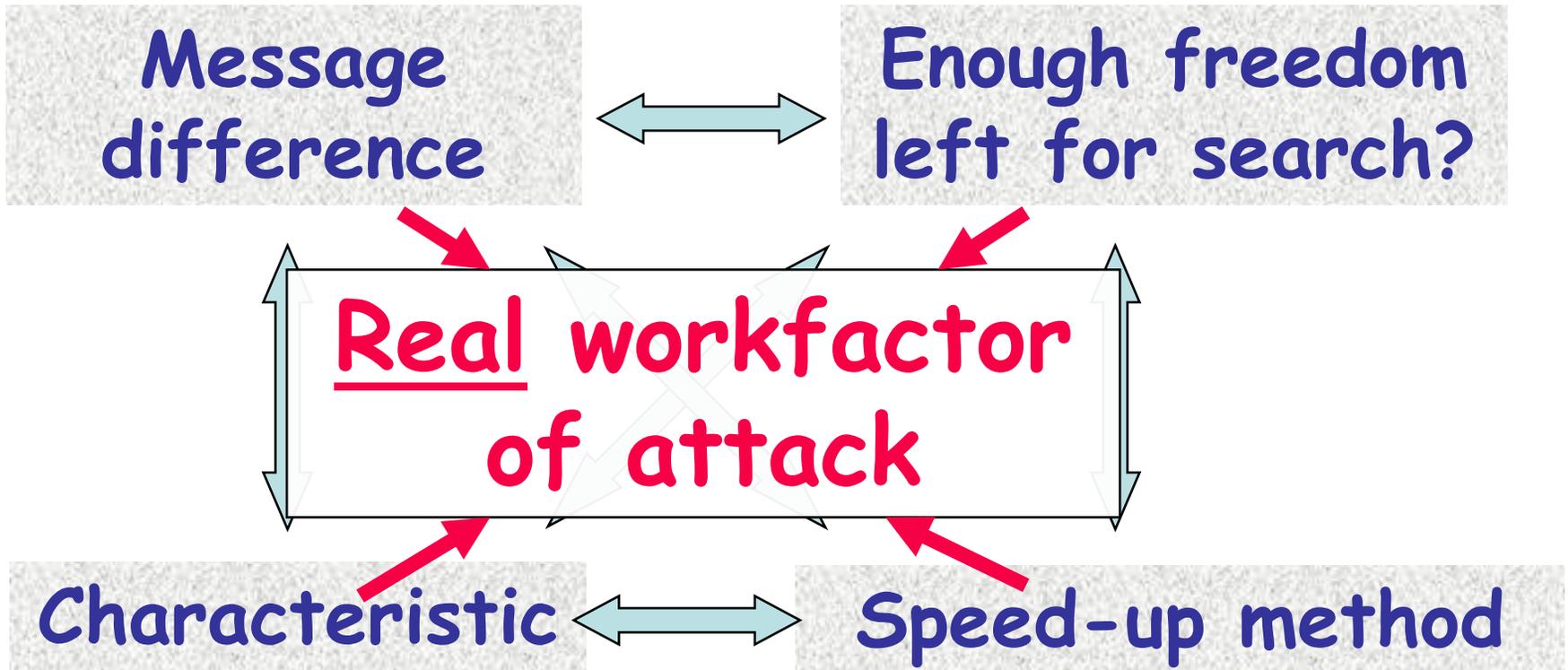
- Results for SHA/SHA-1
 - Up to 25 steps with manual optimization of characteristic (Wang et al., unpublished)
 - Up to 31 steps with especially restricted computer generated characteristic (ongoing work)

- Boundary: we need to leave some degrees of freedom for last phase

Problem of optimization



Problem of optimization



Some results

- SHA

- 1998: 2^{61} [CJ98]
- 2004: 2^{51} [BCJ+05]
- 2005: 2^{39} [WYY05a]
- 2006: 2^{36} [N+06]

- SHA-1

- 2005: 58 steps 2^{33} [WYY05b]
- 2006: 64 steps 2^{35} [DR06]
- 2007: 70 steps 2^{44} [DMR07]

Meaningful Collisions - Motivation

- Setting: Collisions for a hash function can be constructed
- Cryptanalyst perspective: Some more interesting things to find out w.r.t. collision resistance?
 - Constructing collisions **faster**
 - Finding and exploiting degrees of freedom to construct (partially) **meaningful** collisions

Practically relevant if hash function is widely deployed

Color Code

 Under control, attacker can freely choose → **meaningful**

 Not under direct control, determined by the collision search algorithm → **not meaningful**

Meaningful Collisions: Challenges for MD4-style Hash Functions

I. One Commonly Chosen Prefix

II. One Commonly Chosen Prefix +
Partial Control over Colliding Blocks

III. Two Arbitrary Different Chosen Prefixes

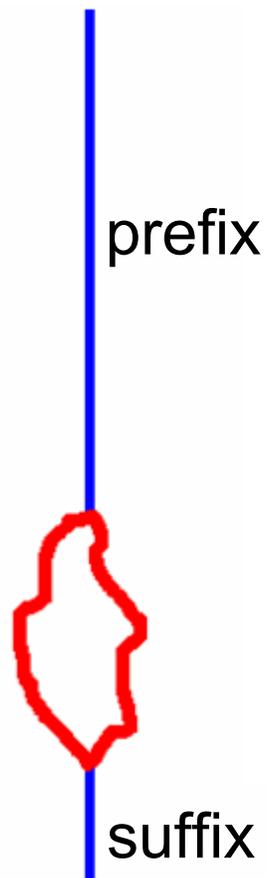
IV. Two Arbitrary Different Chosen Prefixes +
Partial Control over Colliding Blocks



“easier”

“harder”

I. One Commonly Chosen Prefix



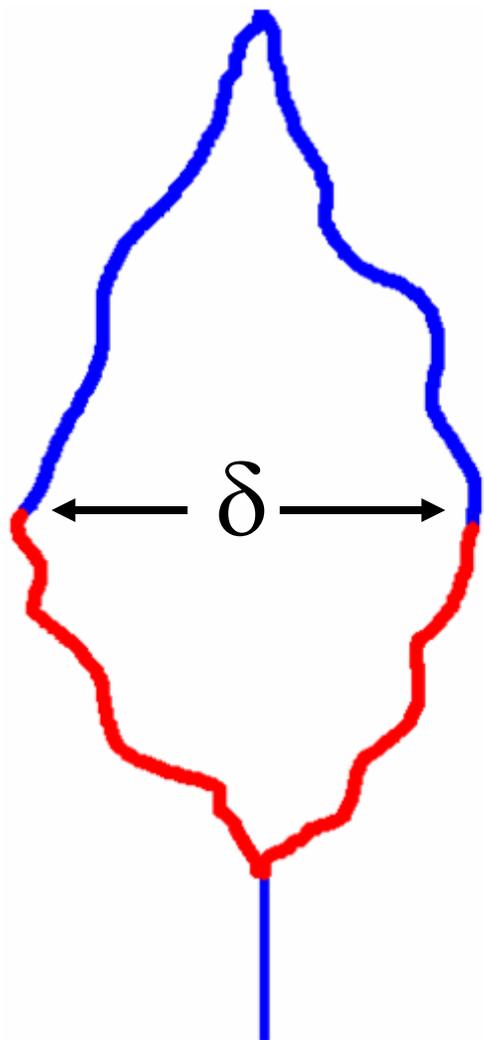
- Small number of colliding blocks
- Enough for colliding meaningful postscript files, etc... (see demo of Lars)

II. One Commonly Chosen Prefix + Partial Control over Colliding Blocks



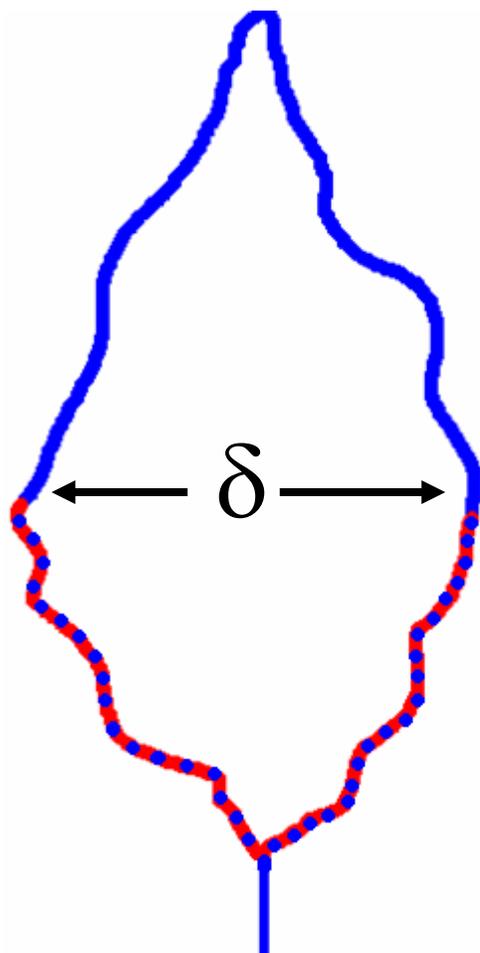
- Small number of colliding blocks
- Application in areas where format restrictions apply
- Example for 64-step SHA-1 in [DR06b]

III. Two Arbitrary Different Chosen Prefixes



- Using feed-forward operation, iteratively cancel out chaining differences with selected near-collision paths
- Usually much more than two message blocks needed
- Speedup: birthday phase before
- Example for MD5: see [SLdW07]

IV. Two Arbitrary Different Chosen Prefixes + Partial Control over Colliding Blocks



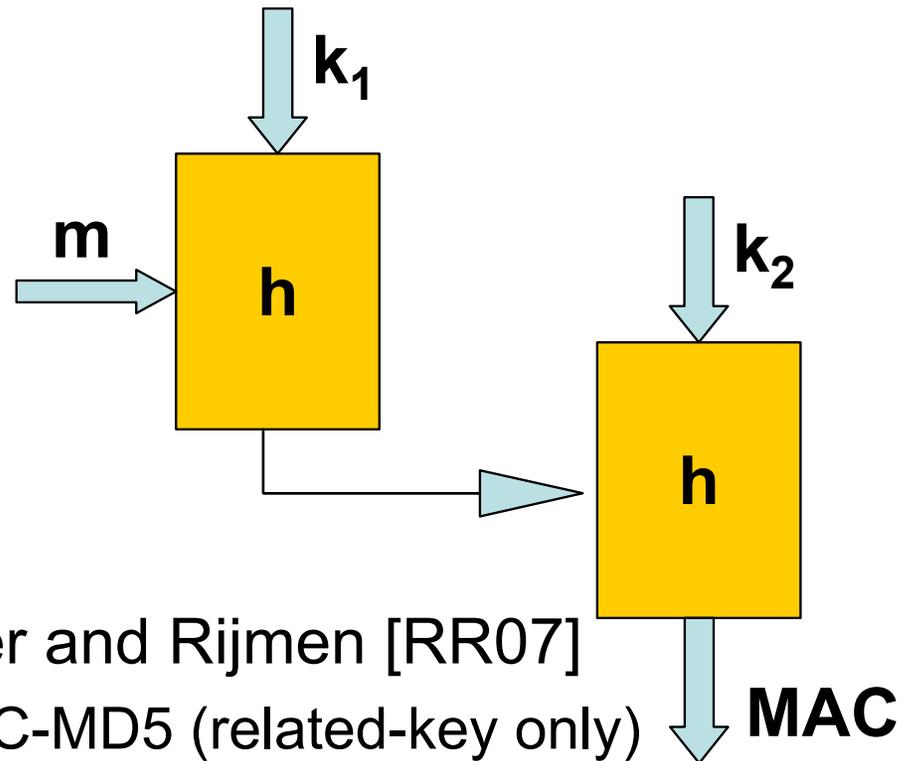
- Using feed-forward operation, iteratively cancel out chaining differences with selected near-collision paths
- Combination of methods

Even that is possible: Example Characteristic for type IV

| i | ∇A_i | ∇W_i |
|-----|-----------------------------------|-----------------------------------|
| -4: | 0000uuu1nu001n1u100nn111u1nn00u1 | } δ |
| -3: | 01000n0n110nunnnun1nu00n1u1n1un00 | |
| -2: | 0uu0nn10uu1nunu10111n01uu1u11n1n | |
| -1: | 1u1n1111110n11n110101n1u1n001001 | |
| 0: | 01u0nu1unu0n01010n10001un0n0n00u | |
| 1: | u001uuu1u011n1nn1001un0nu0u10n1u | n0n0000000000000000000000000000nn |
| 2: | uuu0nu0uuu0nu1uuu11u0001n0u0001u | 10n01000100011000000110110un0011 |
| 3: | 101nn110n1un00nn1uu-0un1uu0-10-1 | 0uu11111010100-----0 |
| 4: | 1011u01u00n11111n000u0-n0100011n | nnn11010101011-10-----1--u1n1u1 |
| 5: | 00n111uu101111nn1u0u10u0-1n00010 | 00u00101100110-----100--n1001un |
| 6: | 0nn01n0nn0-1uu--01n1-11u0--u0n0n | x1un010010-110--011--0101u-11-10 |
| 7: | n0-nu-0110n0--1101-0u10-00-011nu | xu-n0-11-----0-00-0-----x-u--uu |
| 8: | 00n10001n0u10u101--u0n01u1n--0u1 | xu1u010-----1-----1---x---u0 |
| 9: | -11111n---100n-10----0n0u0001--- | -1n0-----0----1-----1-0---- |
| 10: | 0--n1-1-0-010n-0--u--1-01u1n0--- | -nn-----0-----uu--u- |
| 11: | 1---110-0--11n-----1---0-01nu0- | -nu-----n---uu |
| 12: | --nu-1n-n-n--uun--n-1--nu-u1u010 | --n-----1-n----- |
| 13: | u---01--0-0-0--100--1-----0-10 | xnu-----u---u- |
| 14: | x-0-11--1-1-1--011--1-----0-1-1- | -nn-----n- |
| 15: | ----- | x-----u |

Attacks on NMAC/HMAC

Earlier results by
 Kim et al. [KBPH06]
 Contini and Yin [CY06]

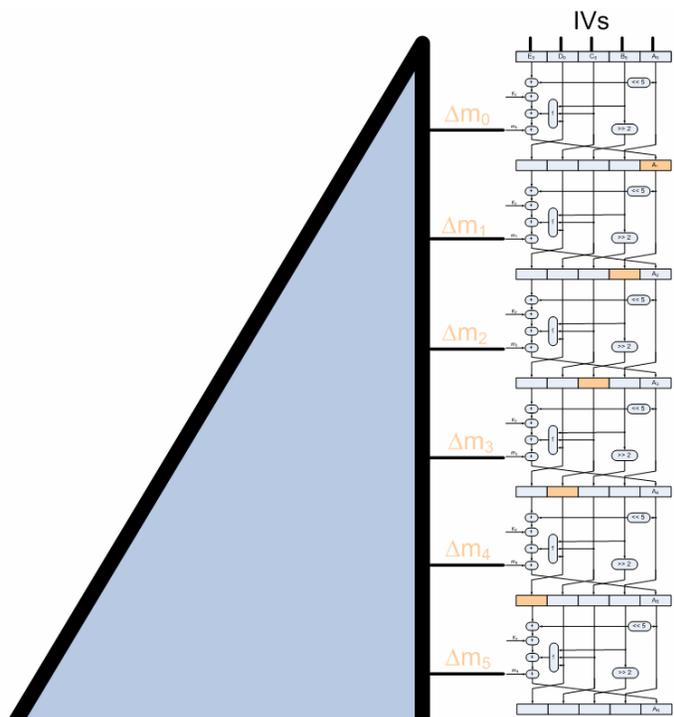


Recent Results by Rechberger and Rijmen [RR07]

- **Full** Key-Recovery for NMAC-MD5 (related-key only)
- **Full** Key-Recovery for NMAC-SHA-1 for 34/80 steps
- Distinguisher and Partial-Key recovery for NMAC/HMAC-SHA-1 for up to **62**/80 steps

What about SHA-2 members?

Probabilities of local collisions

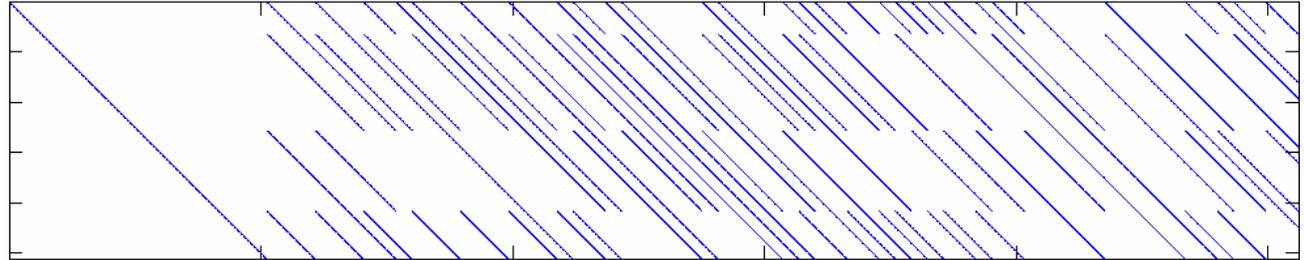


SHA/SHA-1: 2^{-2} to 2^{-5}

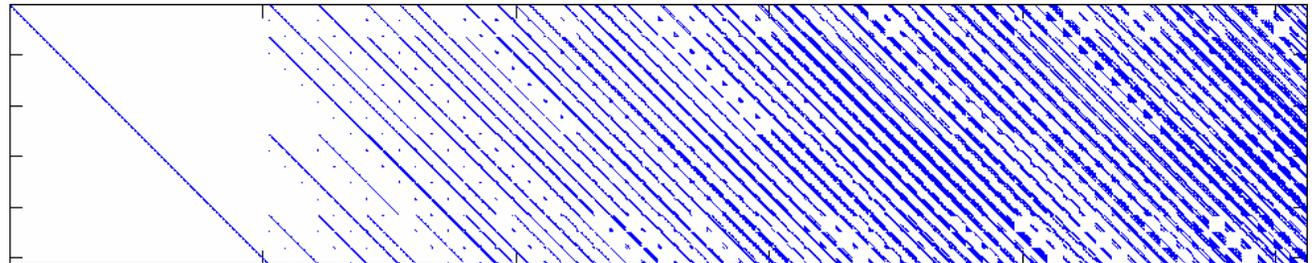
SHA-2: 2^{-38} to 2^{-41}

Comparing the message expansions of the SHA family

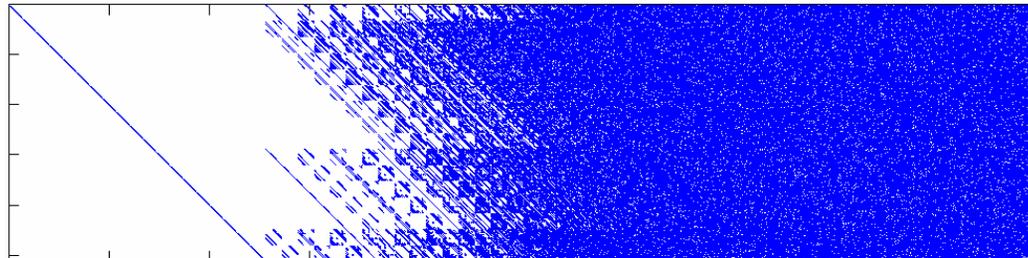
SHA



SHA-1



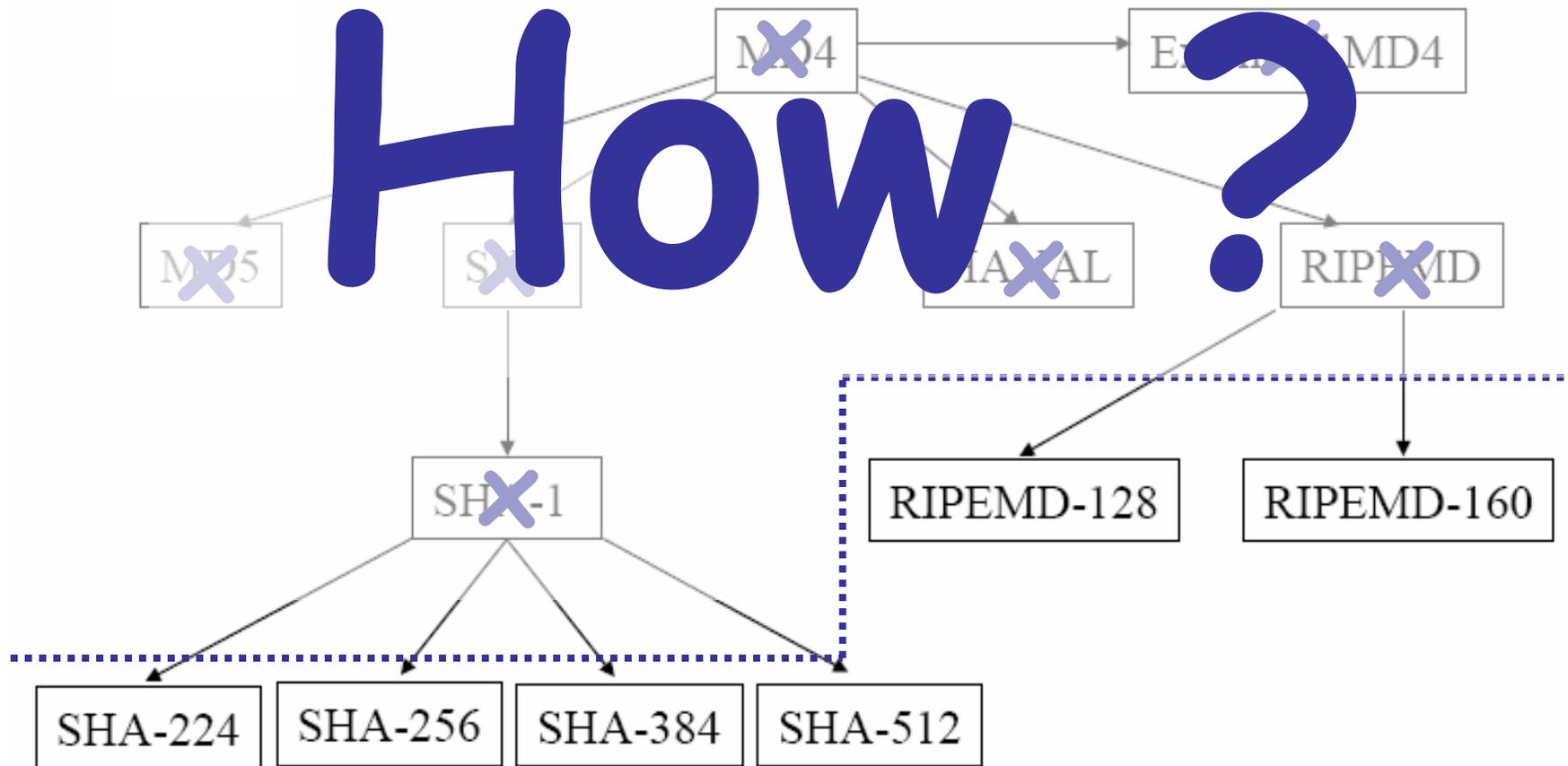
SHA-256
(linearized)



What about SHA-2 members?

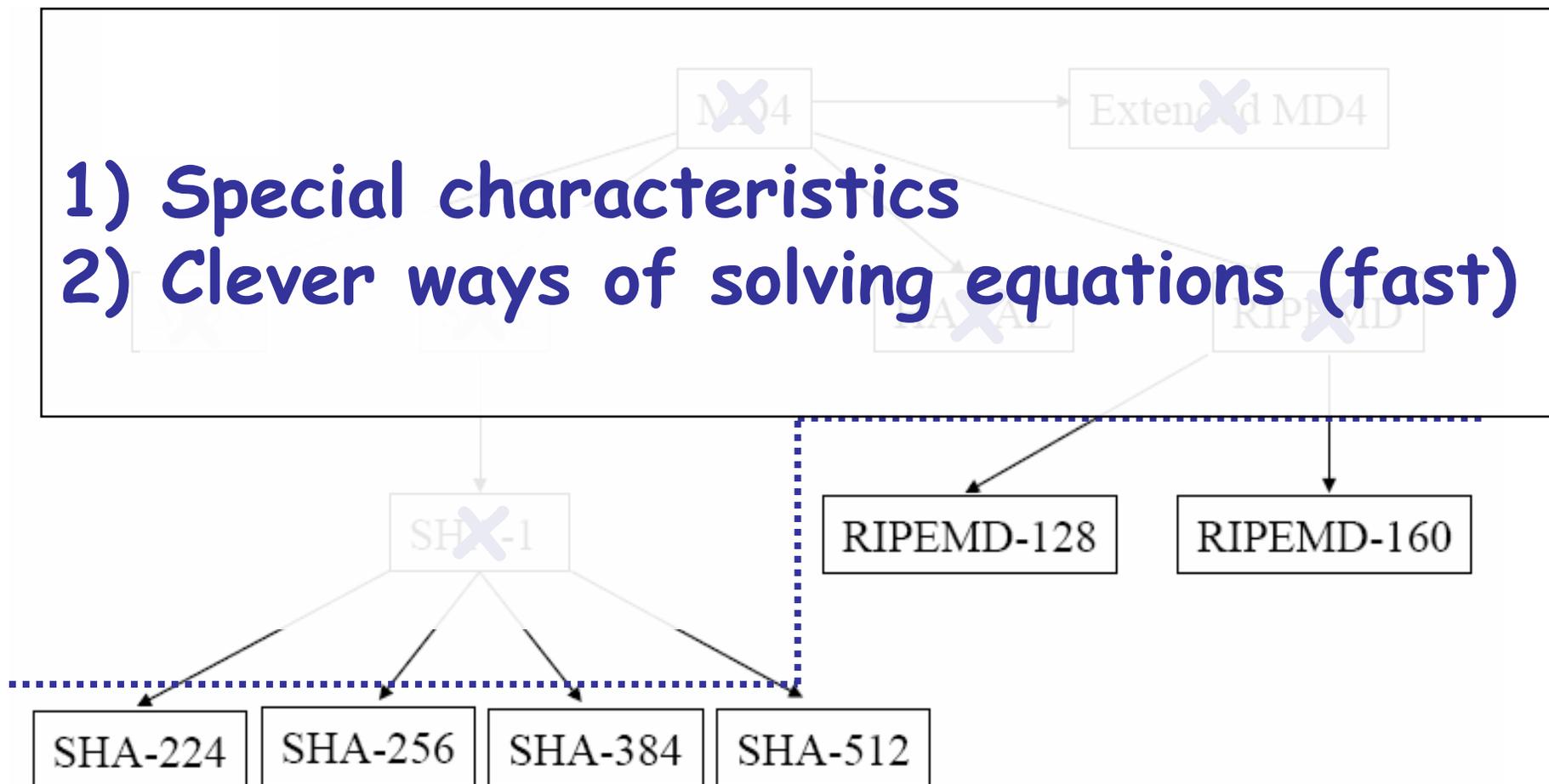
- Collision characteristic for 19-step SHA-224 [MPRR06]
- All publicly known attacks on SHA/SHA-1 since 1997 are not directly applicable to any SHA-2 member
- Message modification more difficult – cyclic dependencies
- New methods for characteristic search not applied yet

Conclusions



Conclusions

- 1) Special characteristics
- 2) Clever ways of solving equations (fast)



Conclusions

- Collisions for MD4, MD5, SHA
- Collision for full (80-step) SHA-1 is getting closer
- Optimization is ongoing
 - 2006: $2^{62} * x$ (unpublished work, estimates)
 - Advanced techniques as used for partial meaningful collisions can also be turned into **faster** collision search
 - 2007: ?

- Apply new insights to other hash functions like RIPEMD-160, SHA-2, new proposals?
- Improved Attacks on NMAC/HMAC?
- Results on (2nd-)preimage resistance?

Some References for MD4/MD5

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- [WY05] Wang, Yu, “How to Break MD5 and Other Hash Functions”, EUROCRYPT 2005

Some References for SHA/SHA-1/SHA-256

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- [BCJ+05] Biham, Chen, Joux et al.: “Collisions of SHA-0 and Reduced SHA-1”, EUROCRYPT 2005
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- [RO05] Rijmen, Oswald: “Update on SHA-1”, CT-RSA 2005
- [WYY05a] Wang, Yu, Yin: “Efficient Collision Search Attacks on SHA-0”, CRYPTO 2005
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Dedicated Attacks on NMAC/HMAC

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Credits

Some slides are by curtesy of

Christophe De Cannière
Florian Mendel
and Vincent Rijmen

**Not enough?
Want more?**

Hash Function Workshop



Barcelona, May 24-25, 2007

events.iaik.tugraz.at/hashworkshop07

Dedicated Attacks on Popular Hash Functions

Q&A

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